ABSORPTION OF SECOND SOUND IN ROTATING HELIUM II

E. M. LIFSHITZ and L. P. PITAEVSKII

Institute for Physical Problems, Academy of Sciences, U.S.S.R.

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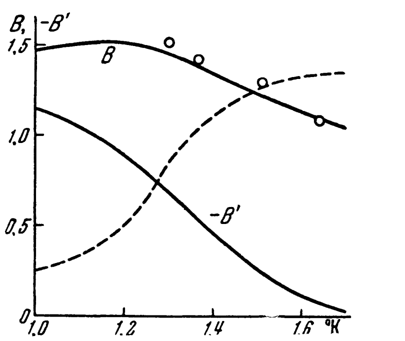
IN a very interesting work Hall and Vinen[1,2] have carried out an experimental investigation of the absorption of second-sound oscillations in rotating helium II, and have also given a qualitative theory for this effect, based on the ideas of Onsager and Feynman concerning the excitation of vortices in helium II when it is rotated.*

A calculation of the "mutual friction" force between the superfluid and normal-fluid components requires first of all an investigation of the interaction of the elementary excitations (rotons, at temperatures which are not excessively low) with the vortex filaments. Besides this it is necessary to take account of hydrodynamic effects not connected with specific properties of the elementary excitations (entrainment of the normal fluid by a vortex filament, and motion of a vortex relative to the superfluid). While Hall and Vinen's investigation of the latter does not give rise to any doubts at the moment, their calculation of the scattering of rotons by vortices does not seem to be correct. Their calculation was carried out in the Born approximation, whereas for the process in question the conditions are actually closer to the opposite limiting case — the quasiclassical approximation. In view of this we are inclined to regard the satisfactory qualitative agreement with experiment which Hall and Vinen obtained as to a certain extent fortuitous (it should also be noted that an undetermined empirical constant figured in their calculation).



We have computed the scattering of rotons by vortices in the quasiclassical approximation. The following result is obtained for the force acting per unit length of filament:†

$$\mathbf{f} = D(\mathbf{v}_R - \mathbf{v}_L) + D'[\boldsymbol{\omega}, \mathbf{v}_R - \mathbf{v}_L]/\omega, \quad D \approx 1.2\varkappa\rho_n\sqrt{\mu kT}/p_0, \quad D' = \varkappa\rho_n \tag{1}$$

(where the symbols have the same meaning as in Refs. 1 and 2). The circulation about the vortex is taken to be $\kappa = 2\pi\hbar/m$; other (multiple) values seem unlikely from a theoretical point of view. In comparison with (1), Hall and Vinen's value of D is about 10/T times larger, and their $D' = 0$ by virtue of the properties of the Born approximation.

Using (1) and taking account of the above-mentioned hydrodynamic effects, as did Hall and Vinen,[2] we calculate the total mutual friction force F_{sn} as a function of $v_s - v_n$, and obtain for their coefficients B and B′ [Eq. (11) of Ref. 2].

$$B = \frac{2\rho}{\rho_n\rho_s\varkappa}\left(\frac{1}{E} + \frac{D}{D^2+D'^2}\right)\Big/\left[\left(\frac{1}{E} + \frac{D}{D^2+D'^2}\right)^2 + \left(\frac{D'}{D^2+D'^2} - \frac{1}{\varkappa\rho_s}\right)^2\right]; \tag{2}$$

$$\frac{B'}{B} = \left(\frac{1}{\varkappa\rho_s} - \frac{D'}{D^2+D'^2}\right)\Big/\left(\frac{1}{E} + \frac{D}{D^2+D'^2}\right) \tag{3}$$

[for $D' = 0$ these expressions reduce to Eqs. (12) and (13) of Ref. 2].

Understandably we can make a comparison with experimental data only for a temperature region in which it is possible to speak of a "gas" of rotons, and their mean free path is small compared with the distances between vortex lines. Values of B calculated from Eqs. (1) and (2) are shown in the figure (dotted line); we see that at the higher temperatures they agree well with Hall and Vinen's measurements, but at 1.3°K they are already much too small.

*We take advantage of the opportunity to observe that the "laminar" model of rotating helium II investigated in Ref. 3 is thermodynamically less favorable than the "vortex" model, and therefore it ought to be discarded.

†A calculation shows that the contribution of the phonons to the friction force becomes important only at temperatures on the order of 0.5°K.

This circumstance apparently points to a more complicated character for the interaction of rotons with vortices which cannot be reduced to simple scattering (in a $\mathbf{p}\cdot\mathbf{v}_s$ field). Apparently, for rotons passing by a vortex at small distances from its axis, processes having the character of "strong interactions" take place, which bring about a momentum transfer of the order of the total momentum of the roton. Several processes can take part in the creation of this interaction, such as the transition of a roton to a "bound" state and subsequently its emission back to a free state (for $\mathbf{p}\cdot\mathbf{v}_s < 0$ the field in which the roton finds itself looks like a field of attraction, and a finite motion in it is possible with an energy $\epsilon < 0$; a roton can go over into such an orbit with the simultaneous excitation of vortex oscillations, and can leave by absorbing the energy of these oscillations.* The "eroding" of a vortex due to its natural oscillations can also play an important role. An investigation of all these effects, however, is very involved, and to a certain extent becomes indeterminate because all of the characteristic lengths turn out to be comparable with atomic distances.

It is natural to try to describe this interaction phenomenologically by introducing a temperature-independent effective vortex diameter ("width") a, corresponding to the transfer of the total momentum of a roton to a vortex. The result is the addition of a term $0.7\,a\rho_n\sqrt{kT/\mu}$ in D. This process makes no contribution to D′; on the contrary one has to introduce a factor $\varphi(a) < 1$ in D′ to account for the corresponding "cut-off" of the integral over the range of distances in the calculation of this quantity. Satisfactory agreement with Hall and Vinen's measurements (shown by circles in the figure) is obtained for $a \cong 10$ A (with $\varphi(a) = 0.6$).† This value is somewhat larger than one would naturally expect. It should be emphasized, however, that it is extremely sensitive to the choice of the experimental values of B. In connection with this situation it would seem to be very desirable to carry out further measurements, particularly at lower temperatures.

Values of B′ calculated from Eq. (3) (for the indicated value of a) are also shown in the figure; it should be pointed out that they are very sensitive with repect to the value of the factor $\varphi(a)$. It would also be desirable to obtain experimental data for this quantity (there are none at present).

We express our thanks to Academician L. D. Landau for a discussion of the problems under investigation.

[1] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. 238A, 204 (1956)
[2] H. E. Hall and W. F. Vinen, Proc. Roy. Soc. 238A, 215 (1956)
[3] L. D. Landau and E. M. Lifshitz, Dokl. Akad. Nauk S.S.S.R. 100, 669 (1955)

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ON THE CONSERVATION OF COMBINED PARITY ‡

V. G. SOLOV'EV

Joint Institute for Nuclear Research

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It has been shown by experiments of Wu and others that in the weak interactions (of the type of β-decay) the parity I is not conserved. It has also been shown that in the weak interactions there are two possibilities: either I, C, and T are separately not conserved, and only their product is conserved, or there is

*It can be shown that this process ought not to lead to the appearance of a large longitudinal friction force (along ω). The point is that the longitudinal component of the momentum lost by the roton is transerrred to quanta of oscillations of the vortex filaments, belonging to the normal fluid, and consequently transfers of momentum from the normal fluid to the superfluid do not take place.

†The value of this factor depends essentially on the method of "cut-off," that is, on the location of the segment a relative to the axis of the vortex. Ths value of φ given above corresponds to placing this segment off to one side of the axis, where the attaction of a roton takes place.

‡ Presented at the Seminar of the Laboratory of Theoretical Physics of the Joint Institute for Nuclear Research, April 20, 1957.