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HYDRODYNAMICAL INTERPRETATION OF ONE CHARACTERISTIC OF LARGE SHOWERS RECORDED IN PHOTOGRAPHIC EMULSIONS

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The experimental distribution of the transverse momentum components of secondary particles is compared with the predictions of the hydrodynamical theory of multiple particle production. It is found that the predictions of the one-dimensional theory for a final temperature of $T_{fin} = mc^2/k$ (where m is the π-meson mass) agree satisfactorily with the experimental data. This permits us to draw some conclusions concerning the nature of $\pi - \pi$ interaction.

THE comparison of predictions of the various phenomenological theories of multiple particle production with experimental results has a decisive value for the confirmation of their validity. Such a comparison made at first for energies of $\gtrsim 10^{12}$ ev (see Ref. 1) indicated that the hydrodynamical theory proposed by Landau describes qualitatively all the known properties of multiple production processes. In the comparison of the theory with experimental data in Ref. 1, however, additional assumptions were made which, without regard to their nature, caused a certain undecisiveness of the conclusion. It was for instance assumed that in the collisions between nucleons and heavy nuclei there exists a system of coordinates in which the disintegration of the matter is symmetric; for the description of the collision a model was used according to which the particle interacts with a cylindrical volume of nuclear matter; the analysis of extensive showers was carried out under the assumption of the independence of the nuclear interaction cross-section from energy, etc.

Taking all the above into account, it is necessary, for a direct confirmation of the theory, to consider such characteristics of the collisions which do not require any complementary assumptions, the correctness of which would be difficult to ascertain. One of such characteristics is the distribution of the transverse momentum components $p_\perp$ of the secondary particles (i.e., components perpendicular to the shower axis). Distribution of the values of $p_\perp$ can be, strictly speaking, found only from the solution of three-dimensional equations, which is a difficult and so far not investigated problem. Special estimates indicate, however, that a hydrodynamical disintegration of particles is, in its main features, determined by a one-dimensional stage[3] and it can be therefore assumed that the distribution of the values of $p_\perp$ is determined solely by thermal motion. This means that we neglect transverse momentum components due to hydrodynamical motion.

In such approximation we shall find the distribution of transverse momenta using the well-known expression for particle density dN in the momentum interval $dp_x dp_y dp_z$

$$dN = g\,(2\pi\hbar c)^{-3}\,[e^{(\epsilon-\mu)/kT} \pm 1]^{-1}\, dp_x dp_y dp_z, \qquad (1)$$

where g is the number of internal degrees of freedom of a particle, ϵ is its energy, and μ is the chemical potential which, according to the thermodynamical conception, we shall set equal to zero. The minus sign corresponds to bosons, and the plus sign to fermions. After integration over p_z we obtain the distribution of transverse components:

$$dN_1 = \frac{g}{(2\pi\hbar c)^3}\, mc\, dp_y dp_z \int_0^\infty \frac{dx}{\exp\left(z\sqrt{x^2+\rho^2+1}\right) \pm 1}, \qquad (2)$$

where $x = p/mc$; $z = mc^2/kT$; $\rho^2 = (p_y^2 + p_z^2)/mc^2$.

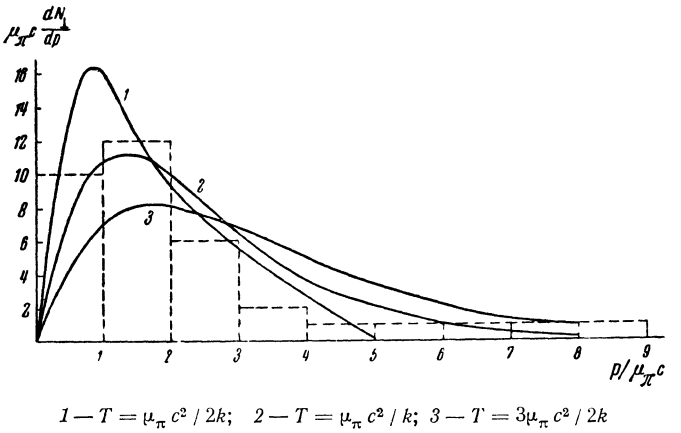
By means of the relation

$$\int_0^\infty \frac{dx}{\exp\left(z\sqrt{x^2+\rho^2+1}\right) \pm 1} = \sqrt{\rho^2+1} \sum_{n=1}^{\infty} (\mp)^{n-1} K_1\left(nz\sqrt{\rho^2+1}\right) \qquad (3)$$

(where K_1 is the Bessel function of imaginary variable) and by integrating over the angle in cylindrical coordinates, we finally obtain

$$dN_\perp = \frac{q}{2\pi^2}\left(\frac{mc}{\hbar}\right)^3 \sum_{n=1}^{\infty} (\mp)^{n-1} K_1\left(n\frac{mc^2}{kT}\sqrt{\left(\frac{p_\perp}{mc}\right)^2+1}\right)\frac{p_\perp dp_\perp}{(mc)^2}. \qquad (4)$$

Theoretical distributions of the transverse components, calculated for bosons for various values of T according to Eq. (4), are shown by the solid curves in the figure. The dotted histogram represents the experimental distributions of $p_\perp$ obtained from the analysis of a single shower recorded in emulsion.[4] Thanks to the unusual path length of secondary particles in the emulsion (16.8 cm) the authors of Ref. 4 were able to determine from scattering measurements the total momentum of the majority of particles (35 out of 39). This permitted us to calculate $p_\perp$ by a simple computation.



$1 - T = \mu_\pi c^2/2k$; $2 - T = \mu_\pi c^2/k$; $3 - T = 3\mu_\pi c^2/2k$

A very similar distribution of the transverse components was obtained also from the measurement of the total momentum of secondary π^0-mesons by means of the pairs associated with them.[5] Distribution of transverse momenta in that case was, too, characterized by a maximum in the region $1 - 2\mu_\pi c^2$ (where μ_π is the π-meson mass).

Several important conclusions may be drawn from the comparison of the experimental and theoretical curves:

1. The one-dimensional theory describes satisfactorily the character of the distribution of transverse momentum components.*

2. Best agreement of both distributions is obtained† for $T = \mu_\pi c^2/k$. The values $T = \mu_\pi c^2/2k$ and $T = 3\mu_\pi c^2/2k$ are already difficult to reconcile with experimental data, although the scarcity of the latter does not permit to rule these values out. The value $T = \mu_\pi c^2/k$ is in agreement with previous indications, based on the analysis of other experimental results (for composition of showers see Refs. 6 and 7 and for energy spectrum see Ref. 3).

3. Since the value of temperature is connected with the value of the interaction cross-section of secondary particles[6] (evidently π-mesons) we can conclude that in the order of magnitude the latter equals the geometrical cross-section of the nucleon $(\hbar/\mu_\pi c)^2$.

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ON THE RELATION BETWEEN "ACCIDENTAL" DEGENERACY AND "HIDDEN" SYMMETRY OF A SYSTEM

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An example is discussed which confirms the connection between "accidental" degeneracy and "hidden" symmetry of a system. The symmetry of a two-dimensional oscillator is studied, and the relation is found between the quantization of the oscillator and that of a certain operator of the type of an angular momentum.

SUPPOSE the operator H is invariant with respect to some group G of transformations. Then the application of these transformations results in the expression of the eigenfunctions of the operator H belonging to an eigenvalue E_n in terms of each other, and thus gives a certain representation D_n of the group G.[1] As

*The cited calculations have shown that the introduction of the conical stage[2] of hydrodynamical motion predicts that the mean value of $p_\perp$ is $\gtrsim Mc$ (where M is the nucleonic mass), contradicting the histogram shown in the figure. It is possible that the conical stage of burst is necessary for the description of particle interaction at considerably higher energies than the energy of the shower of Ref. 4 ($\sim 5 \times 10^{12} - 10^{13}$ ev).

†It should be noted that since we neglected the possible influence of the hydrodynamical transverse components the given values of T are, strictly speaking, the upper limits of the values.