

with the tabular data for Eu<sup>146</sup>. On the basis of measurements of this isotope from the time of its separation from the gadolinium fraction we evaluated the period of the parent substance Gd<sup>146</sup> to be  $12 \pm 4$  hours. It should be noted that the mass number of Gd<sup>146</sup> was determined with the same degree of reliability as that of the daughter europium isotope, which belongs, according to Seaborg's<sup>2</sup> tables, in class C (mass number "reliable or probable").

<sup>1</sup> Gorodinskii, Pokrovskii, Preobrazhenskii, Murin, and Titov, Dokl. Akad. Nauk, SSSR 112, 405 (1957); Soviet Phys. "Doklady" 2, 39 (1957).

<sup>2</sup> Seaborg, Perlman, and Hollander, Table of Isotopes, (M., 1956).

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### Possibility of Constructing a Chain of Equations for Model Operators

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THE THEORY OF MODEL TRANSFORMATIONS<sup>1</sup> is characterized by the fact that the model operator  $M_n$  transforming the model state

$$|\varphi_1 \dots \varphi_n\rangle = \prod_{\gamma=1}^n \varphi(\gamma)$$

into the real state of the system  $|\Psi\rangle = \Psi(1 \dots n)$ , is an operator function of all the dynamic variables of the system. To reduce the many-particle problem to a single-particle problem, let us introduce the sequence of generalized transition amplitudes

$$\langle \varphi_1 \dots \varphi_n | \Psi \rangle; \langle \varphi_1 \dots \varphi_{\alpha-1}, \varphi_{\alpha+1} \dots \varphi_n | \Psi \rangle \equiv \langle \dots (\varphi_\alpha) \dots | \Psi \rangle, \dots, \langle \varphi_\alpha \varphi_\beta | \Psi \rangle; \langle \varphi_\alpha | \Psi \rangle; |\Psi\rangle,$$

where, for example,

$$\langle \dots (\varphi_\alpha) \dots | \Psi \rangle = \int \frac{d\tau}{d\tau_\alpha} \prod_{\gamma \neq \alpha}^+ \varphi(\gamma) \Psi(1 \dots n).$$

Assuming that the real and model states of the system are described by the wave equations

$$i\partial_t |\Psi\rangle = \left\{ \sum_\alpha T(\alpha) + \sum_{\alpha\beta} H(\alpha\beta) \right\} |\Psi\rangle, \quad i\partial_t \varphi_\alpha = \{T(\alpha) + U(\alpha)\} \varphi_\alpha,$$

we obtain a system of equations

$$i\partial_t \langle \varphi_1 \dots \varphi_n | \Psi \rangle = \sum_{\alpha\beta} \langle \varphi_\alpha \varphi_\beta | H(\alpha\beta) | \dots (\varphi_\alpha \varphi_\beta) \dots | \Psi \rangle - \sum_\alpha \langle \varphi_\alpha | U(\alpha) | \dots (\varphi_\alpha) \dots | \Psi \rangle,$$

$$\dots \dots \dots$$

$$\left\{ i\partial_t - \sum_{\alpha \neq \gamma} T(\alpha) - \sum_{\alpha\beta \neq \gamma} H(\alpha\beta) \right\} \langle \varphi_\gamma | \Psi \rangle = \langle \varphi_\gamma | \sum_{\alpha \neq \gamma} H(\alpha\gamma) - U(\gamma) | \Psi \rangle,$$

similar to the system of equations for a density matrix.<sup>2</sup> In the stationary case the system assumes the form

$$\begin{aligned}
\left\{ E - \sum_{\alpha} E_{\alpha} \right\} \langle E_1 \dots E_n | E \rangle &= \sum_{\alpha\beta} \langle E_{\alpha} E_{\beta} | H(\alpha\beta) | \langle \dots (E_{\alpha} E_{\beta}) \dots | E \rangle \rangle - \sum_{\alpha} \langle E_{\alpha} | U(\alpha) | \langle \dots (E_{\alpha}) \dots | E \rangle \rangle; \\
\left\{ E - \sum_{\alpha \neq \gamma} E_{\alpha} - T(\gamma) \right\} \langle \dots (E_{\gamma}) \dots | E \rangle &= \sum_{\alpha\beta \neq \gamma} \langle E_{\alpha} E_{\beta} | H(\alpha\beta) | \langle \dots (E_{\alpha} E_{\beta} E_{\gamma}) \dots | E \rangle \rangle \\
&\quad + \sum_{\alpha \neq \gamma} \langle E_{\alpha} | H(\gamma\alpha) - U(\alpha) | \langle \dots (E_{\alpha} E_{\gamma}) \dots | E \rangle \rangle; \\
\dots; \\
\left\{ E - E_{\gamma} - \sum_{\alpha \neq \gamma} T(\alpha) - \sum_{\alpha\beta \neq \gamma} H(\alpha\beta) \right\} \langle E_{\gamma} | E \rangle &= \langle E_{\gamma} | \sum_{\alpha \neq \gamma} H(\alpha\gamma) - U(\gamma) | E \rangle.
\end{aligned} \tag{1}$$

System (1) jointly with the equations

$$\left\{ E - \sum_{\alpha} T(\alpha) - \sum_{\alpha\beta} H(\alpha\beta) \right\} | E \rangle = 0, \quad \{ E_{\alpha} - T(\alpha) - U(\alpha) \} | E_{\alpha} \rangle = 0$$

determines the single-particle potential  $U$  on the surface of constant energy  $(E - \sum_{\alpha} E_{\alpha} = 0)$  and is equivalent to the model transformation  $| E \rangle = M_n | E_1 \dots E_n \rangle$ . The last assertion becomes obvious, if we introduce the sequence of model operators

$$\begin{aligned}
\langle \dots (E_{\alpha}) \dots | E \rangle &= M_1(\alpha) | E_{\alpha} \rangle; \\
\langle \dots (E_{\alpha} E_{\beta}) \dots | E \rangle &= M_2(\alpha\beta) | E_{\alpha} E_{\beta} \rangle; \\
\dots; \\
\langle E_{\alpha} | E \rangle &= M_{n-1}(\dots \alpha - 1, \alpha + 1, \dots) | E_1, \dots, E_{\alpha-1} E_{\alpha+1}, \dots, E_n \rangle; \\
| E \rangle &= M_n | E_1 \dots E_n \rangle
\end{aligned}$$

and find the operator equations for  $M_p$ .

Consideration of stationary transitions on a constant energy surface corresponds in this case to examination of such single-particle states which satisfy the requirement  $\partial_t \langle \varphi_1 \dots \varphi_n | \Psi \rangle = 0$ , that is an analogue of the usual condition of normalization of  $\partial_t \langle \Psi | \Psi \rangle = 0$ .

<sup>1</sup>R. I. Eden and N. C. Francis, Phys. Rev. 97, 1366 (1955).

<sup>2</sup>N. N. Bogoliubov, Lectures on Quantum Statistics, Kiev, 1949.

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## Some Remarks on Slow Processes of Transformation of Elementary Particles

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**A**S IS KNOWN, there are two types of slow processes:

(a) lepton:

$$\begin{aligned}
n \rightarrow e + \bar{\nu} + p, \quad \mu \rightarrow e + \nu + \bar{\nu}, \quad \mu + p \rightarrow n + \nu, \quad \pi \rightarrow \mu + \nu, \\
K \rightarrow \mu + \nu, \quad K \rightarrow \mu + \nu + \pi, \quad K \rightarrow e + \nu + \pi,
\end{aligned}$$

(b) non-lepton:

$$K \rightarrow 2\pi, \quad K \rightarrow 3\pi, \quad \Lambda(\Sigma) \rightarrow N + \pi, \quad \Xi \rightarrow \Lambda + \pi.$$

The constants of the interactions responsible for these processes in units of  $\hbar = \mu = c = 1$  (where  $\mu$  is the  $\pi$ -meson mass) are of the same order of magnitude as  $G^2 = 10^{-14} - 10^{-12}$ . This suggests that the same mechanism (for example, the universal Fermi interaction<sup>1</sup>) may lie at the basis of all\* these processes. This idea is supported by the fact that

\*Non-conservation of parity in the decay of hyperons, although it has not yet been proved experimentally, almost inescapably follows from the established parity non-conservation in the decay of  $K$ -mesons.