[8] A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) 29, 246 (1955); Soviet Phys. JETP 29, 161 (1956); 30, 601 (1956); Soviet Phys. JETP 3, 634 (1956); 30, 990 (1956); Soviet Phys. JETP 3, 783 (1956); V. M. Maksimenko and A. I. Nikishov, J. Exptl. Theoret. Phys. (U.S.S.R.) 31, 727 (1956); Soviet Phys. JETP 4, 614 (1957).

[9] L. Landau and E. Lifshitz, *Statistical Physics*, GITTL (1951), § 75.

[10] G. E. Uhlenbeck and E. Beth, Physica 3, 729 (1936); 4, 915 (1937).

[11] R. H. Milburn, Rev. Mod. Phys. 27, 1 (1955); I. L. Rozental', J. Exptl. Theoret. Phys. 28, 118 (1955); Soviet Phys. JETP 1, 166 (1955).

[12] F. J. Dyson, Phys. Rev. 99, 1037 (1955); G. Takeda, Phys. Rev. 100, 440 (1955).

[13] J. Orear, Phys. Rev. 100, 288 (1955).

[14] J. S. Kovacs, Phys. Rev. 101, 397 (1956).

Translated by M. A. Melkanoff
237

SOVIET PHYSICS JETP VOLUME 5, NUMBER 5 DECEMBER, 1957

Contribution to the Theory of Transport Processes in a Plasma Located in a Magnetic Field

E. S. FRADKIN

P. N. Lebedev Institute of Physics, Academy of Sciences, U.S.S.R.

(Submitted to JETP editor July 5, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 1176-1187 (May, 1957)

The mean statistical characteristics (velocity, heat current, stress tensor, etc.) have been determined for transport phenomena in a plasma located in a magnetic field.

THE PRESENT ARTICLE develops the theory of a plasma located in electric and magnetic fields (a brief account of the principal results of the author's analysis[1] is presented here). A method of approximations is used to solve the Boltzmann equation. The essence of this method lies in computing the terms of the distribution function, expanded in powers of some small parameter characteristic of the specific problem. One thus finds a so-called "local" distribution function which must be understood in the following sense: the complete distribution function is a function of 6 variables (3 coordinates and 3 velocity components) while the "local" distribution function depends explicitly upon the velocity only, and its dependence upon the coordinates enters only through the externally-acting forces and the mean statistical characteristics (temperature, density, and in general, velocity of the center of mass), which play the role of parameters. The dependence of these characteristics upon the coordinates is obtained by solving a certain system of equations which also arise from the Boltzmann equation. These equations represent a generalization of the well-known hydrodynamical equations.

1. As is well known[2], the Boltzmann equation for particles of type s, mixed with particles of other types, has the following form in the presence of magnetic and electric fields (the notation is that used by Chapman and Cowling):

$$\frac{\partial f_s}{\partial t} + \mathbf{v}_i \frac{\partial f_s}{\partial x_i} + \frac{e_s}{m_s}\left\{\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right\}_i \frac{\partial f_s}{\partial \mathbf{v}_i} = -\sum_{n=1} J_{sn}(f_s, f_n). \tag{1}$$

It will later be found convenient to rewrite the Boltzmann equation (1) in terms of a new set of independent variables; specifically, we transform the velocity of particles of a given type into their velocity with respect to the center of mass. The latter is given by the following formula:

$$\mathbf{v}_0 = \sum_{s=1}^{n} \int m_s \mathbf{v} f_s d\mathbf{v} \Big/ \sum_s \int m_s f_s d\mathbf{v}. \tag{2}$$

Eq. (1) takes the following form in terms of the new variables (henceforth, we write v for v^{rel}):

$$\frac{Df_s}{Dt} + v_i \frac{\partial f_s}{\partial x_i} + \left\{ \frac{e_s}{m_s} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right) - \frac{D\mathbf{v}_0}{Dt} \right\}_i \frac{\partial f_s}{\partial v_i}$$

$$+ \frac{e_s}{m_s c} [\mathbf{v}\mathbf{H}] \frac{\partial f_s}{\partial \mathbf{v}} - \frac{\partial v_s}{\partial v_i} v_k \frac{\partial v_{0i}}{\partial x_k} = - \sum\nolimits_n J_{sn} (f_s, f_n), \quad (3)$$

where $D/Dt = \partial/\partial t + v_{0i}\partial/\partial x_i$.

As an approximate method for solving the system of equations (1), we expand the solution as a power series in a small parameter of the problem. In the case of axial symmetry, the following parameters are available: L, the dimension of the system, l, the mean free path, and R, the Larmor radius. Depending upon the relations between these three parameters, the general problem may be divided into three cases: 1) strong magnetic field ($\nu/\omega \ll 1$), where ν is the collision frequency, and ω the Larmor frequency, $R/l \ll 1$; 2) weak magnetic field ($\nu/\omega \gg 1$) $R/l \gg 1$; 3) intermediate region $R/l \sim 1$.

Formally, however, it is convenient to seek the distribution function as a series in the dimensional parameter

$$f = \sum \lambda^m f^{(m)}, \quad (4)$$

where it follows from our exposition that the parameter λ turns out to be $\lambda = 1/\omega$ in a strong magnetic field, $\lambda = 1/\nu$ in a weak magnetic field, and $\lambda = 1/L$ in the intermediate case.

The equations for each term $f^{(m)}$ are easily obtained in the usual fashion. One must also include the fact that the Boltzmann equation must satisfy certain integral conditions of solubility:

$$\int Df_s d\mathbf{v} = 0, \quad (5a)$$

$$\sum_{s=1}^{n} \int Df_s m_s \mathbf{v} d\mathbf{v} = 0, \quad (5b)$$

$$\sum_{s=1}^{n} \int Df_s \frac{m_s v^2}{2} d\mathbf{v} = 0. \quad (5c)$$

These equations are the equations of hydrodynamics, generalized to include a plasma in the presence of magnetic and electric fields, and can be written explicitly as follows:

a) equation of continuity for particles of type s

$$DN_s / Dt + N_s \operatorname{div} \mathbf{v}_0 + \operatorname{div} (N_s \tilde{\mathbf{v}}_s) = 0, \quad (6a)$$

b) equation of motion

$$\rho \frac{D v_{0i}}{Dt} = \sum_{s=1}^{n} e_s N_s \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right)_i - \frac{\partial \Pi_{ik}}{\partial x_k} + \frac{1}{c} [\mathbf{j}\mathbf{H}]_i, \quad (6b)$$

c) equation of conservation of energy for all the particles

$$D\mathcal{E}/Dt + \mathcal{E} \operatorname{div} \mathbf{v}_0 + \operatorname{div} \mathbf{q} + \Pi_{ik} \partial v_{0i} / \partial x_k$$

$$- \mathbf{j} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}\mathbf{H}] \right) = 0, \quad (6c)$$

where we have introduced the following notation:

$$\int f_s d\mathbf{v} = N_s, \quad \int f_s \mathbf{v} d\mathbf{v} = N_s \tilde{v}_s, \quad \mathbf{q} = \sum_{s=1}^{n} \int \frac{m_s v^2}{2} \mathbf{v} f_s d\mathbf{v},$$

$$\Pi_{ik} = \sum_{s=1}^{n} \int m_s v_i v_k f_s d\mathbf{v}, \quad \mathbf{j} = \sum e_s N_s \tilde{\mathbf{v}}_s, \quad \rho = \sum N_s m_s, \quad \mathcal{E} = \sum \frac{m_s v^2}{2} f_s d\mathbf{v}.$$

It will be found convenient in later analyses to be certain that Eqs. (5a) and (5c) are identically satisfied. The following system of equations may easily be shown to fulfill this requirement

$$\frac{Df_s}{Dt} + v_i \frac{\partial f_s}{\partial x_i} + \left\{ \frac{e_s}{m_s} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] - \frac{D\mathbf{v}_0}{Dt} \right\}_i \frac{\partial f_s}{\partial v_i} - \frac{\partial f_s}{\partial v_i} v_k \frac{\partial v_{0i}}{\partial x_k}$$

$$+ \frac{e_s}{m_s c} [\mathbf{v}\mathbf{H}] \frac{\partial f_s}{\partial \mathbf{v}} - \sum_{r=1}^{n} \left\{ \frac{DN_r}{Dt} + N_r \operatorname{div} \mathbf{v}_0 + \operatorname{div} (N_r \tilde{\mathbf{v}}_r) \right\} \frac{\partial f_s}{\partial N_r}$$

$$- \left(2/3 \sum_{r=1}^{n} N_r k \right) \left\{ \frac{3}{2} \sum_{r=1}^{n} N_r k \frac{DT}{Dt} - \frac{3}{2} kT \sum_{r=1}^{n} \operatorname{div} (N_r \tilde{\mathbf{v}}_r) + \operatorname{div} \mathbf{q} \right. \quad (7)$$

$$\left. + \Pi_{ik} \partial v_{0i} / \partial x_k - \mathbf{j} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right) \right\} \partial f_s / \partial T = - \sum_n J_{sn} (f_s, f_n),$$

where instead of using the total interval energy $\mathcal{E}$ of the plasma, we have introduced the "effective" temperature of the plasma (the word "effective" will be omitted hereinafter), *i.e.*,

$$\mathcal{E} = \frac{3}{2} \sum_{s=1}^{n} N_s kT.$$

Note that in expanding the mean quantities in terms of a small parameter, it may be assumed that the zero approximation completely determines N_s and T.

2. We shall now consider a stationary problem with axial symmetry, a mixture of two charged gases in a strong magnetic field. Let N_1, $-e$, m, and f_1 be the density, charge, mass, and distribution function of the electrons, and N_2, Ze, M, and f_2 be the corresponding properties of the ions.

In this case the system of equations (1) has the form:

$$v_{0x} \frac{\partial f_1}{\partial x} + v_x \frac{\partial f_1}{\partial x} - \left\{ \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right) + v_{0x} \frac{\partial \mathbf{v}_0}{\partial x} \right\} \frac{\partial f_1}{\partial \mathbf{v}} -$$

$$- \frac{\partial f_1}{\partial v_i} v_x \frac{\partial v_{0i}}{\partial x} - \frac{e}{mc} [\mathbf{vH}] \frac{\partial f_1}{\partial \mathbf{v}} = - \{ J_{11}(f_1, f_1) + J_{12}(f_1, f_2) \}, \tag{8}$$

$$v_{0x} \frac{\partial f_1}{\partial x} + v_x \frac{\partial f_2}{\partial x} + \left\{ \frac{eZ}{M} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] - v_{0x} \frac{\partial \mathbf{v}_0}{\partial x} \right\} \frac{\partial f_2}{\partial \mathbf{v}} -$$

$$- \frac{\partial f_2}{\partial v_i} v_x \frac{\partial v_{0i}}{\partial x} + \frac{eZ}{Mc} [\mathbf{vH}] \frac{\partial f_2}{\partial \mathbf{v}} = - \{ J_{22}(f_2, f_2) + J_{21}(f_2, f_1) \}, \tag{9}$$

where the z-axis is taken along $\mathbf{H}$.*

In this case the gradients of the concentration and the electric field have no z-components, and if the system is sufficiently large in the z-direction, $L_z \gg l$, the distribution will be Maxwellian, and we shall assume that $v_{0z} = 0$.

Expanding the distribution function and the mean quantities (except N_2 and T) as series in the parameter $1/\omega$ and equating coefficients of equal power, we obtain equations which may be used for computing successive approximations to the distribution function. We thus obtain the following equations for $f_s^{(0)}$ and $f_s^{(1)}$

$$- \frac{e}{mc} [\mathbf{vH}] \frac{\partial f_1^{(0)}}{\partial \mathbf{v}} = 0, \quad \frac{eZ}{Mc} [\mathbf{vH}] \frac{\partial f_0^{(2)}}{\partial \mathbf{v}} = 0, \tag{10}$$

$$- \frac{e}{mc} [\mathbf{vH}] \frac{\partial f_1^{(1)}}{\partial \mathbf{v}} + v_x \frac{\partial f_1^{(0)}}{\partial x} - \left\{ \frac{e}{m} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right) \right\} \frac{\partial f_1^{(0)}}{\partial \mathbf{v}} = - \{ J_{11} + J_{12} \}, \tag{11}$$

$$\frac{eZ}{Mc} [\mathbf{vH}] \frac{\partial f_2^{(1)}}{\partial \mathbf{v}} + v_x \frac{\partial f_2^{(0)}}{\partial x} + \left\{ \frac{eZ}{M} \left(\mathbf{E} + \frac{1}{c} [\mathbf{v}_0 \mathbf{H}] \right) \right\} \frac{\partial f_2^{(0)}}{\partial \mathbf{v}} = - \{ J_{22} + J_{21} \}. \tag{12}$$

All the equations which are used for computing successive approximations may be symbolically written as follows:

$$[\mathbf{v}\boldsymbol{\omega}_s] \, \partial f_s^{(m)} / \partial \mathbf{v} = F_s(v, x); \quad \boldsymbol{\omega}_s = (eZ_s/m_s c) \, \mathbf{H}. \tag{13a}$$

It is evident that the solution to this equation makes sense only when the result of the integration does not depend on the contour of integration. In order for this to be true, it is sufficient that the following relation hold:

$$\operatorname{curl} \{ [\mathbf{u}\boldsymbol{\omega}_s] F_s(u, x) / [\mathbf{u}, \boldsymbol{\omega}_s]^2 \} = 0. \tag{13b}$$

* Since the problem is solved locally, we replace the cylindrical coordinates with cartesian coordinates having the x-axis along the radius r and the axis $x_2 = y$ along the θ-direction.

The significance of condition (13b) becomes clearer when the solution to Eq. (13) is written in terms of new variables

$$f_s^{(m)} = \frac{1}{\omega_s}\int_0^\alpha F_s(\mathbf{v}, x)\, d\alpha. \quad \cot\alpha = v_x/v_y. \quad (13c)$$

Condition (13b) becomes in these variables a periodicity condition, $f_s^{(m)}(\alpha) = f_s^{(m)}(\alpha + 2\pi)$, for which it is necessary and sufficient that the following hold:

$$\int_0^{2\pi} F_s(v, x)\, d\alpha = 0. \quad (13d)$$

Condition (13d) together with the requirement that the temperature and density be wholly obtainable from the zero approximation, completely determine the solution to the equations for $f_s^{(m)}$. Computations yield the following results

$$f_n^{(0)} = N_n \pi^{-3/2}\beta_n^3 \exp(-\beta_n^2 v^2),\ \beta_1 = (m/2kT)^{1/2},$$

$$\beta_2 = (M/2kT)^{1/2}$$

$$f_1^{(1)} = \frac{1}{\omega_{\mathrm{el}}}[\mathbf{A}_1\mathbf{n}_0]\,\mathbf{v} f_1^{(0)};\ f_2^{(1)} = -\frac{1}{\omega_{\mathrm{ion}}}[\mathbf{A}_2\mathbf{n}_0]\,\mathbf{v} f_2^{(0)}, \quad (14)$$

$$\mathbf{A}_n = \frac{\nabla N_n}{N_n} + \frac{\nabla T}{T} + \frac{e\mathbf{E}}{kT} + \frac{e}{ckT}[\mathbf{v}_0^{(1)}\mathbf{H}] - \left(\frac{5}{2} - \beta_n v^2\right)\frac{\nabla T}{T};$$

$$n_0 = \mathbf{H}/H;\ \omega_{\mathrm{el}} = eH/mc;\ \omega_{\mathrm{ion}} = eZH/Mc. \quad (15)$$

The relative velocities $\mathbf{v}_n$ and heat currents $\mathbf{q}_n$ for each type of particle are found to be

$$\tilde{\mathbf{v}}_1^{(1)} = \frac{kT}{m\omega_{\mathrm{el}}}\left[\left(\frac{\nabla N_1}{N_1} + \frac{\nabla T}{T} + \frac{e\mathbf{E}}{kT} + \frac{1}{\omega_{\mathrm{el}}}[\mathbf{v}_0\mathbf{n}_0]\right), \mathbf{n}_0\right], \quad (16)$$

$$\tilde{\mathbf{v}}_2^{(1)} = -\frac{kT}{M\omega_{\mathrm{ion}}}\left[\left(\frac{\nabla N_2}{N_2} + \frac{\nabla T}{T} + \frac{e\mathbf{E}}{kT} - \frac{1}{\omega_{\mathrm{ion}}}[\mathbf{v}_0\mathbf{n}_0]\right), \mathbf{n}_0\right], \quad (17)$$

$$\mathbf{v}_0^{(1)} = -\frac{kT}{M\omega_{\mathrm{ion}}}\left[\frac{\frac{\nabla N_2}{N_2} + \frac{\nabla T}{T} - \frac{eZ\mathbf{E}}{kT} - \frac{N_1 mZ}{N_2 M}\left(\frac{\nabla N_1}{N_1} + \frac{\nabla T}{T} + \frac{e\mathbf{E}}{kT}\right)}{1 + N_1 m_1/N_2 M}, \mathbf{n}_0\right], \quad (18)$$

$$\mathbf{q}_1^{(1)} = \frac{5}{2}\frac{N_1(kT)^2}{m\omega_{\mathrm{el}}}\left[\frac{\nabla T}{T}\mathbf{n}_0\right] + \frac{5}{2}N_1 kT\tilde{\mathbf{v}}_2, \quad (19)$$

$$\mathbf{q}_{\mathrm{tot}}^{(1)} = \mathbf{q}_1^{(1)} + \mathbf{q}_2^{(1)} = \frac{5N_1(kT)^2}{2m\omega_{\mathrm{el}}}\left[\frac{\nabla T}{T}\mathbf{n}_0\right]\left(1 - \frac{N_2}{N_1 Z}\right) + \frac{5}{2}kT(N_1\tilde{\mathbf{v}}_2 + N_2\tilde{\mathbf{v}}_2). \quad (20)$$

In this approximation, the addition to the stress tensor Π_{ik} is zero. Since a complete solution of the problem requires knowledge of the addition to the stress tensor, it is necessary to obtain a higher approximation to the distribution function. Instead of solving the Boltzmann equation in higher approximations, it is more convenient to use it for finding a system of equations for the mean quantities of interest, and solve these equations directly. Thus, obtaining in the second approximation the equations for $\tilde{v}_n^{(2)}$, $q_n^{(2)}$ and $\Pi_{ik}^{(2)}$, and solving them (to the order m/M), we find

$$\tilde{\mathbf{v}}_{0,x}^{(2)} = -\frac{N_1 kT\nu_{\mathrm{el}}}{N_2 Zm\omega_{\mathrm{el}}^2}\left\{\frac{N_1'}{N_1} + \frac{N_2'}{ZN_2} + \frac{T'}{T}\left(\frac{1}{Z} - \frac{1}{2}\right)\right\},\quad \tilde{v}_1^{(2)} = -\frac{N_2 M}{N_1 m}\tilde{v}_2^{(2)}, \quad (21)$$

$$\tilde{v}_{1x} = -\frac{\nu_{\mathrm{el}}kT}{m\omega_{\mathrm{el}}^2}\left(1 - \frac{N_1}{N_2 Z}\right)\left\{\frac{N_2'}{N_2 Z} + \frac{N_1'}{N_1} + \frac{T'}{T}\left(\frac{1}{Z} - \frac{1}{2}\right)\right\}, \quad (22)$$

$$q_{1x}^{(2)} = -\frac{N_1(kT)^2\nu_{\mathrm{el}}}{m\omega_{\mathrm{el}}^2}\left\{\left(\frac{7}{4} - \frac{3}{2Z} + \sqrt{2}\frac{N_1}{N_2 Z^2}\right)\frac{T'}{T} - 1.5\left(\frac{N_1'}{N_1} + \frac{N_2'}{N_2 Z}\right)\right\} + \frac{5}{2}N_1 kT\tilde{v}_{2x}, \quad (23)$$

$$q_{\text{tot}\,x}^{(2)} = -\frac{N_2 (kT)^2 \nu_{\text{el}}}{m\omega_{\text{el}}^2}\left\{\left(\sqrt{2}\left(\frac{M}{m}\right)^{1/2} + \frac{7N_1}{4N_2} - \frac{3N_1}{2N_2 Z} + \frac{15N_1}{2N_2 Z^2} + \frac{\sqrt{2}N_1^2}{Z^2 N_2^2}\right)\frac{T'}{T}\right.$$

$$\left. + 1.5\left(\frac{N_1'}{N_1} + \frac{N_2'}{ZN_2}\right)\frac{N_1}{N_2}\right\} + \frac{5}{2} N_1 kT \tilde{v}_{1x}^{(2)}, \tag{24}$$

$$\Pi_{xx}^{(2)} = -\frac{N_2}{2}\frac{(kT)^2}{M\omega_{\text{ion}}^2}\left\{\frac{N_2''}{N_2} + 1.2\frac{T''}{T} + 0.8\frac{N_2'T'}{N_2 T} - \frac{eE'}{kT} - \left(\frac{N_2'}{N_2}\right)^2\right.$$

$$\left. + 0.9\left(\frac{T'}{T}\right)^2 - \frac{H'}{H}\left(\frac{N_2'}{N_2} + 1.2\frac{T'}{T} - \frac{eE}{kT}\right)\right\}, \tag{25}$$

$$\Pi_{yy}^{(2)} = \frac{N_2 (kT)^2}{2M\omega_{\text{ion}}}\left\{\frac{N_2''}{N_2} + \frac{2.8T''}{T} + 3.2\frac{N_2'T'}{N_2 T} - \frac{eE'}{kT} - \left(\frac{N_2'}{N_2}\right)^2\right.$$

$$\left. + 1.1\left(\frac{T'}{T}\right)^2 - \frac{H'}{H}\left(\frac{N_2'}{N_2} + 3.8\frac{T'}{T} - \frac{eE}{kT}\right)\right\}, \tag{26}$$

$$\Pi_{zz} = -\Pi_{xx} - \Pi_{yy}, \tag{27}$$

where

$$N' = \frac{dN}{dx},\quad N'' = \frac{d^2N}{dx^2},\quad \text{etc.} \qquad \nu_{\text{el}} = \frac{4\sqrt{2\pi}}{3m^{1/2}} N_2 \frac{(Ze^2)^2}{(kT)^{3/2}} \ln\left(\frac{3(kT)^{3/2}}{2e^3\pi^{1/2}(N_1+N_2)^{1/2}}\right).$$

The remaining components are zero in this approximation (note that the expression for Π_{ik} is obtained in the approximation $Z \ll (M/m)^{1/2}$. As for Π_{xy}, it only starts to differ from zero in the third approximation and is found to be

$$\Pi_{xy}^{(3)} = -\frac{\sqrt{\pi}8eZc^3(kT)^{1/2}}{15H^3} N_2^2 M^{3/2} \ln\left(\frac{3(kT)^{3/2}}{2e^3\pi^{1/2}(N_1+N_2)^{1/2}}\right)\left\{\frac{7N_2'T'}{2N_2 T} + \frac{47T''}{16T}\right.$$
$$\left. - \frac{7}{8}\left(\frac{T'}{T}\right)^2 - \frac{47H'T'}{16HT} + \frac{3}{4}\left[\frac{N_2''}{N_2} - \left(\frac{N_2'}{N_2}\right)^2 - \frac{H'}{H}\left(\frac{N_2'}{N_2} - \frac{eE'}{kT}\right)\right]\right\}. \tag{28}$$

3. In this section we shall investigate the existence of transport phenomena in a mixture of electrons and ions, for a stationary case with arbitrary symmetry, when $\nu/\omega \sim 1$. The expansion parameters are in this case R/L and l/L. Formally, this leads to the fact that the collision integral and the magnetic term $\mathbf{v} \times \mathbf{H}\,\partial f/\partial \mathbf{v}$ must be considered on equal footing in the Boltzmann equation. The zero approximation for f_n is obtained from the system of equations

$$+\frac{e}{mc}[\mathbf{vH}]\frac{\partial f_1^{(0)}}{\partial \mathbf{v}} = J_{11}(f_1^{(0)}, f_1^{(0)}) + J_{12}(f_1^{(0)}, f_2^{(0)}),\quad -\frac{eZ}{Mc}[\mathbf{vH}]\frac{\partial f_2^{(0)}}{\partial \mathbf{v}} = J_{22}(f_2^{(0)}, f_2^{(0)}) + J_{21}(f_2^{(0)}, f_1^{(0)}). \tag{29}$$

It is easily seen that solution (14) satisfies the system (29). We also obtain the following system of equations for $f_n^{(1)}$:

$$(\mathbf{Av}) f_1^{(0)} - \frac{e}{mc}[\mathbf{vH}]\frac{\partial f_1^{(1)}}{\partial \mathbf{v}} = -\sum_{n=1}^{2}\{J_{1n}(f_1^{(0)}, f_n^{(0)}) + J_{1n}(f_1^{(0)}, f_n^{(0)}),$$
$$(\mathbf{A_2v}) f_2^{(0)} + \frac{eZ}{Mc}[\mathbf{vH}]\frac{\partial f_2^{(1)}}{\partial \mathbf{v}} = -\sum_{n=1}^{2}\{J_{2n}(f_2^{(0)}, f_n^{(0)}) + J_{2n}(f_2^{(0)}, f_n^{(0)}); \tag{30}$$

$$\mathbf{A}_1 = \frac{\nabla N_1}{N_1} + \frac{\nabla T}{T} + \frac{e\mathbf{E}}{kT} + \frac{e}{ckT}[\mathbf{v}_0^{(1)}\mathbf{H}] - \left(\frac{5}{2} - \beta_1^2 v^2\right)\frac{\nabla T}{T},$$
$$\mathbf{A}_2 = \frac{\nabla N_2}{N_2} + \frac{\nabla T}{T} - \frac{eZ\mathbf{E}}{kT} - \frac{eZ}{ckT}[\mathbf{v}_0^{(1)}\mathbf{H}] - \left(\frac{5}{2} - \beta_1^2 v^2\right)\frac{\nabla T}{T}. \quad (31)$$

Considering the form of the free terms, it is natural to seek solutions of the form $f_n^{(1)} = \mathbf{h}\cdot\mathbf{v} f_n^{(0)}$, and to seek $\mathbf{h}$ in the form of a series of Laguerre polynomials to the $3/2$ power,

$$\mathbf{h}_n = \sum \mathbf{p}_n^{(r)} L_r^{3/2}(\beta_n^2 v^2).$$

From (30), the following system of algebraic equations* may be obtained for the coefficients $\mathbf{p}_n^{(r)}$

$$\mathbf{A}_1^{(1)}\delta_{0s} - \frac{5}{2}\frac{\nabla T}{T}\delta_{1s} - [\boldsymbol{\omega}_{\mathrm{el}}\mathbf{p}_1^{(s)}]\frac{\Gamma(s+5/2)}{\Gamma(s+1)\Gamma(5/2)} + \sum_{r=0}^{\infty}\{\mathbf{p}_1^{(r)}H_{rs}^{\mathrm{I}} + \mathbf{p}_2^{(r)}H_{rs}^{(12)}\} = 0,$$
$$\mathbf{A}_2^{(1)}\delta_{0s} - \frac{5}{2}\frac{\nabla T}{T}\delta_{1s} + [\boldsymbol{\omega}_{\mathrm{ion}}\mathbf{p}_2^{(s)}]\frac{\Gamma(s+5/2)}{\Gamma(s+1)\Gamma(5/2)} + \sum_{r=0}^{\infty}\{\mathbf{p}_2^{(r)}H_{rs}^{\mathrm{II}} + \mathbf{p}_2^{(r)}H_{rs}^{(21)}\} = 0, \quad (32)$$

where $R = N_1/N_2Z^2$,

$$H_{rs}^{I} = \nu_{\mathrm{el}}\begin{vmatrix} 1 & \frac{3}{2} & \frac{15}{8} & \cdots \\ \frac{3}{2} & \frac{13}{4} + \sqrt{2}R & \frac{3\sqrt{2}}{4}R + \frac{69}{16} & \cdots \\ \frac{15}{8} & \frac{3\sqrt{2}}{4}R + \frac{69}{16} & \frac{45\sqrt{2}}{16}R + \frac{433}{64} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix};$$

$$H_{rs}^{(12)} = -\frac{m_1}{M}\nu_{\mathrm{el}}\begin{vmatrix} 1 & \frac{3}{2} & \frac{15}{8} & \cdots \\ 0 & 0 & 0 & \cdots \\ 0 & 0 & 0 & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{vmatrix};$$

$$H_{rs}^{(21)} = -\frac{N_1}{N_2}\nu_{\mathrm{el}}\begin{vmatrix} 1 & 0 & \cdots \\ \frac{3}{2} & 0 & \cdots \\ \frac{15}{8} & 0 & \cdots \\ \cdots & \cdots & \cdots \end{vmatrix};$$

$$H_{rs}^{\mathrm{II}} = \frac{N_1 m}{N_2 M}\nu_{\mathrm{el}}\begin{vmatrix} 1 & 0 & 0 & \cdots \\ 0 & \frac{15}{2} + \sqrt{2}\left(\frac{M}{m}\right)^{1/2}R^{-1}; & \frac{3\sqrt{2}}{4}\left(\frac{M}{m}\right)^{1/2}R^{-1} & \cdots \\ 0 & \frac{3\sqrt{2}}{4}\left(\frac{M}{m}\right)^{1/2}R^{-1}; & \frac{175}{8} + \frac{45\sqrt{2}}{16}\left(\frac{M}{m}\right)^{1/2}R^{-1} & \cdots \end{vmatrix}.$$

*The system of equations (32) was also solved by Landshoff[3], who obtained numerical values of the transport coefficient for certain values of the ratio ω/ν and for $Z = 1, 2, 3$; we are interested in the explicit form of the dependence of the transport coefficient upon Z, M, and ω/ν, as derived below.

It is easily seen that

$$\widetilde{\mathbf{v}}_n^{(1)} = \frac{kT}{M_n}\mathbf{p}_n^{(0)}, \quad \mathbf{q}_n = \frac{5\,(kT)^2 N_n}{2M_n}(\mathbf{p}_n^{(0)} - \mathbf{p}_n^{(1)}).$$

The condition that the sum of the relative momenta be zero yields

$$\mathbf{p}_2^{(0)} = -N_1\mathbf{p}_1^{(0)}/N_2.$$

We shall consider later the case when the concentration gradients are perpendicular to the magnetic field (transport phenomena in the H direction are the same as if there were no magnetic field); in addition we set $N_1 \approx N_2 Z$ (quasineutral gas). Omitting a rather cumbersome computation, we obtain with sufficient accuracy the following expressions for the quantities of interest (we omit the expression for Π_{ik} as being too cumbersome):

$$\widetilde{\mathbf{v}}_1^{(1)} = -\frac{kT}{m\omega_{\mathrm{el}}}\left\{\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla N_1}{N_1}\right)\alpha_1 + \frac{\nabla T}{T}\alpha_2\right\}\left(1 - \frac{N_1}{N_2 Z}\right)$$
$$+ \frac{kT}{m\omega_{\mathrm{el}}}\left[\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla N_1}{N_1} + \frac{\nabla T}{T}\frac{1-Z}{Z}\right), \mathbf{n}_0\right], \mathbf{j} \approx -eN_1\widetilde{\mathbf{v}}_1^{(1)}, \tag{34}$$

$$\widetilde{\mathbf{v}}_0^{(1)} = \frac{kT}{m\omega_{\mathrm{el}}}\left\{\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla N_1}{N_1}\right)\alpha_1 + \frac{\nabla T}{T}\alpha_2 - \left[\left(\alpha_3\left(\frac{\nabla N_1}{N_1} + \frac{\nabla N_2}{ZN_2}\right) + \alpha_4\frac{\nabla T}{T}\right.\right.\right.$$
$$\left.\left.\left. + \frac{1}{Z}\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla T}{T} - \frac{cZ\mathbf{E}}{kT}\right)\right), \mathbf{n}_0\right]\right\}, \tag{35}$$

$$\widetilde{\mathbf{v}}_2^{(1)} = -\widetilde{\mathbf{v}}_1^{(1)} N_1 m/N_2 M, \tag{36}$$

$$\mathbf{q}_{\mathrm{tot}}^{(1)} = \frac{5N_1(kT)^2}{2m\omega_{\mathrm{el}}}\left\{\frac{15}{4a}\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla N_1}{N_2}\right)\left(\frac{\omega}{\nu}\right)_{\mathrm{el}} + \alpha_5\frac{\nabla T}{T} + \left[\left(\alpha_6\frac{\nabla T}{T} - \alpha_7\left(\frac{\nabla N_2}{ZN_2} + \frac{\nabla N_1}{N_1}\right)\right), \mathbf{n}_0\right]\right\}, \tag{37}$$

where

$$a = 10.56 + \frac{9.1}{Z} + \frac{2}{Z^2} + \frac{25}{4}\left(\frac{\omega}{\nu}\right)_{\mathrm{el}}^2, \quad \alpha_1 = -\left(\frac{\nu}{\omega}\right)_{\mathrm{el}}\left(1 - \frac{7.3 + 3.17/Z}{a}\right),$$

$$\alpha_2 = \frac{1+Z}{Z}\alpha_1 + \frac{75}{8a}\left(\frac{\omega}{\nu}\right)_{\mathrm{el}}, \quad \alpha_3 = \frac{5{,}6}{a}, \quad \alpha_4 = -\frac{6{,}6}{a},$$

$$\alpha_5 = -\frac{35}{8a} - \frac{2N_2(\sqrt{2}\,(M/m)^{1/2} + 15/2Z)(\omega/\nu)_{\mathrm{el}}}{5N_1(\omega/\nu)_{\mathrm{el}}^2 + \frac{4}{25}(15N_1/2ZN_2 + (2M/m)^{1/2}Z)^2},$$

$$\alpha_6 = \frac{1}{a}\left\{\frac{5}{2}\left(\frac{\omega}{\nu}\right)_{\mathrm{el}}^2 + \left(4.88 + 2.1\frac{1}{Z}\right)\left(1 + \frac{1}{Z}\right)\right\} + 1 + \frac{1}{Z}$$
$$- \frac{(\omega/\nu)^2 N_2/ZN_1}{(\omega/\nu)_{\mathrm{el}}^2 + \frac{4}{25}(15N_1/2ZN_2 + (2M/m)^{1/2}Z)^2},$$

$$\alpha_7 = 1 + \frac{4.88 + 2.1/Z}{a}, \quad \mathbf{n}_0 = \mathbf{H}/H.$$

4. We shall consider here transport phenomena in a mixture of electrons and ions located in a weak magnetic field, and in order to be more general, we shall consider a non-stationary problem (the center of mass has arbitrary velocity). In this case it is convenient to alter somewhat the system of equations (3) in such a way that the process of successive approximation does not apply to v_0, and that the conditions of solubility (5) are automatically satisfied. In order to do this, it is sufficient to replace $D\mathbf{v}_0/Dt$ in Eq. (3) by the right-hand term of the equation of motion [see (6b)]. Omitting a rather laborious development, we present the final results for this case

$$\tilde{\mathbf{v}}_1^{(1)} = \frac{kT}{m\nu_{\mathrm{el}}} \frac{(4.5R^2+13.35R+3.4)(\nabla N_1/N_1+\nabla T/T+eE/kT+(e/ckT)[\mathbf{v}_0\mathbf{H}])+(5.1+9.9R)\nabla T/T}{4.5R^2+5.4R+1} \quad (38)$$

$$\mathbf{q}_{\mathrm{tot}}^{(1)} = -\frac{N_1(kT)^2}{m\nu_{\mathrm{el}}} \frac{(11.25R^2+43.3R+13.5)\{\nabla N_1/N_1+(e/kT)(\mathbf{E}+c^{-1}[\mathbf{v}_0\mathbf{H}])\}}{4.5R^2+5.4R+1}$$

$$+\frac{(11.25R^2+93.3R+46.5)(\nabla T/T)}{4.5R^2+5.4R+1}. \quad (39)$$

The diffusion coefficient is

$$D = (kT/m\nu_{\mathrm{el}})(4.5R^2+13.35R+5.4)/(4.5R^2+5.41R+1.01), \quad R = N_1/N_2Z^2. \quad (40)$$

The thermal diffusion coefficient is

$$D_T = (kT/m\nu_{\mathrm{el}})(4.5R^2+23.3R+8.5)/(4.5R^2+5.41R+1.01). \quad (41)$$

The electrical conductivity is

$$\sigma_{\mathrm{e}} = De^2N_1/kT. \quad (42)$$

The viscosity is
a) for $(N_1/N_2)(M/m)^{1/2}Z^{-2} \gg 1$

$$\eta_1 = 5(kT)^{5/2}M^{1/2}/8\pi^{1/2}e^4Z^4\ln L; \quad (43)$$

b) for $(N_1/N_2)(M/m)^{1/2}Z^{-2} \ll 1$ (for heavy elements)

$$\eta_2 = 5\sqrt{2}\,(kT)^{5/2}N_1m^{1/2}/8\pi^{1/2}e^4Z^2N_2\ln L;\ \ L = 3\,(kT)^{3/2}/2e^3\pi^{1/2}(N_1+N_2)^{1/2}. \quad (44)$$

Note that in the latter case the Boltzmann equation can be solved exactly without expansion in series of Laguerre polynomials, and yields the following expressions whose values are close to those obtained above: .

$$D^{\mathrm{ex}} = 32kT/3\pi m\nu_{\mathrm{el}},\ D_T = {}^5/_2D^{\mathrm{ex}};\ \eta^{\mathrm{ex}} = 32\sqrt{2}\,m^{1/2}(kT)^{5/2}N_1/15\pi^{3/2}Z^2N_2e^4\ln L. \quad (45)$$

Note, in conclusion, that in the case of strong temperature gradients, it is not enough to restrict oneself to the dependence of the stress tensor Π_{ik} on the gradients of v_0, and for heavy elements we find

$$\Pi_{ik} = (N_1+N_2)kT\delta_{ik} - \eta_2\left(\frac{\partial v_{0i}}{\partial x_k}+\frac{\partial v_{0k}}{\partial x_i}-\frac{2}{3}\operatorname{div}\mathbf{v}_0\delta_{ik}\right)+\frac{2}{5N_1kT}\left(\frac{\partial q_i}{\partial x_k}+\frac{\partial q_k}{\partial x_i}-\frac{2}{3}\operatorname{div}\mathbf{q}\delta_{ik}\right). \quad (46)$$

5. a) Of special interest is the case when the plasma is located in a strong magnetic field deviating slightly from axial symmetry, *i.e.*, when

$$\frac{R}{L}\frac{l}{L}\frac{\Delta H}{H} \ll 1$$

(ΔH is the variation in the magnetic field due to deviation from axial symmetry). In this case we obtain the following final results:

$$\mathbf{q}_{\text{tot}} = \frac{5N_2(kT)^2}{2m\omega_{\text{el}}}\left[\left(\frac{\nabla N_1}{N_1}+\frac{\nabla N_2}{ZN_2}+\frac{\nabla T}{T}\left(2+\frac{1}{Z}-\frac{N_2}{N_1Z}\right)\right),\ \mathbf{n}_0\right]$$

$$-\frac{kT}{M(\nu\omega)_{\text{ion}}}[(0.8\nabla\operatorname{div}\mathbf{q}_2^{(1)}+1.80\nabla\operatorname{div}\mathbf{v}_0),\ \mathbf{n}_0]$$

$$+\frac{e}{m\omega_{\text{el}}\cdot\nu_{\text{ion}}}\left[(\mathbf{E}+\frac{1}{c}[\mathbf{v}_0^{(1)}\mathbf{H}]),\mathbf{n}_0\right](0.46\operatorname{div}\mathbf{q}_2^{(1)}+0.64\operatorname{div}\mathbf{v}_0^{(1)})$$

$$-\frac{N_2(kT)^2\nu_{\text{el}}}{m\omega_{\text{el}}^2}\frac{\nabla T}{T}\left\{\sqrt{2}\left(\frac{M}{m}\right)^{1/2}+\frac{7N_1}{4N_2}-\frac{3N_1}{2ZN_2}+\frac{15N_1}{2N_2Z^2}+\frac{\sqrt{2}N_1^2}{N_2^2Z^2}\right\}, \tag{47}$$

$$\tilde{\mathbf{v}}_1 = \frac{kT}{m\omega_{\text{el}}}\left[\left(\frac{\nabla N_1}{N_1}+\frac{\nabla N_2}{ZN_2}+\frac{\nabla T}{T}\left(1+\frac{1}{Z}\right)\right),\ \mathbf{n}_0\right]-$$

$$-\left(\frac{\nu}{\omega^2}\right)_{\text{el}}\frac{kT}{m}\left\{\frac{\nabla N_1}{N_1}+\frac{\nabla N_2}{ZN_2}+\frac{\nabla T}{T}\left(\frac{1}{Z}-\frac{1}{2}\right)\right\}\left(1-\frac{N_1}{N_2Z}\right)$$

$$-\frac{[(0.46\,\nabla\operatorname{div}\mathbf{q}_2^{(1)}+0.64\,\nabla\operatorname{div}\mathbf{v}_0^{(1)}),\ \mathbf{n}_0]}{N_2M(\omega\nu)_{\text{ion}}}, \tag{48}$$

$$\mathbf{v}_0 = -\frac{kT}{M\omega_{\text{ion}}}\left[\left(\frac{\nabla N_2}{N_2}+\frac{\nabla T}{T}-\frac{eZ\mathbf{E}}{kT}\right),\ \mathbf{n}_0\right]-\frac{\nu_{\text{el}}N_1kT}{N_2Zm\omega_{\text{el}}^2}\left\{\frac{\nabla N_1}{N_1}+\frac{\nabla N_2}{ZN_2}\right.$$

$$\left.+\frac{\nabla T}{T}\left(\frac{1}{Z}-\frac{1}{2}\right)\right\}+\frac{[(0.46\,\nabla\operatorname{div}\mathbf{q}_2^{(1)}+0.64\,\nabla\operatorname{div}\mathbf{v}_0^{(1)}),\ \mathbf{n}_0]}{N_2M(\omega\nu)_{\text{ion}}}, \tag{49}$$

where

$$\mathbf{v}_0^{(1)} = -\frac{kT}{M\omega_{\text{ion}}}\left[\left(\frac{\nabla N_2}{N_2}+\frac{\nabla T}{T}-\frac{eZ\mathbf{E}}{kT}\right),\ \mathbf{n}_0\right],\quad \mathbf{q}_2^{(1)} = -\frac{5}{2}\frac{N_2(kT)^2}{M\omega_{\text{ion}}}\left[\frac{\nabla T}{T}\,\mathbf{n}_0\right], \tag{50}$$

$$\nabla = \mathbf{i}\partial/\partial x+\mathbf{j}\partial/\partial y,\quad \nu_{\text{ion}} = 8\pi^{1/2}N_2e^4Z^4\ln L/3\,(kT)^{3/2}M^{1/2}.$$

b) To conclude this section, we shall consider the case of a plasma consisting of electrons and ions in the presence of an electromagnetic field which varies in space (both in magnitude and direction) and in time. We shall restrict ourselves to the case when the time variations of all the quantities that characterize the plasma during the periods between collisions are smaller than the quantities themselves (*i.e.*, $\partial f/\partial t \ll \nu f$ where ν is the collision frequency. In this case (as in Sec. 4), it is convenient, in order to obtain the local distribution function, to transform the Boltzmann equation in such a way as to satisfy identically the conditions of solubility (5), independently of the values of N, T, or $\mathbf{v}_0$. Here the space and time variation of these quantities need not be expanded in power series in a small parameter, but their values will be obtained by solving the complete system of generalized hydrodynamical equations. Omitting the computations, we present here the final results. In this case:

$$\tilde{\mathbf{v}}_1^{(1)} = \frac{kT}{m\nu_{\text{el}}}\left\{\alpha_1[[\mathbf{A}_1\mathbf{n}_0]\mathbf{n}_0]+\alpha_2\left[\frac{\nabla T}{T}\,\mathbf{n}_0\right]+\alpha_3[\mathbf{A}_1\mathbf{n}_0]\right.$$

$$\left.+\alpha_4\left[\frac{\nabla T}{T}\,\mathbf{n}_0\right]+\alpha_5\frac{\nabla T}{T}+\alpha_6\mathbf{A}_2\right\};\quad \mathbf{j}^{(1)} = -eN_1\tilde{\mathbf{v}}_1^{(1)}, \tag{51}$$

$$\mathbf{q}_{\text{tot}} = \frac{5N_1(kT)^2}{2m\nu_{\text{el}}}\left\{\beta_1\left[[\mathbf{A}_1\mathbf{n}_0]\,\mathbf{n}_0\right]+\beta_2\left[\left[\frac{\nabla T}{T}\,\mathbf{n}_0\right]\mathbf{n}_0\right]+\beta_3[\mathbf{A}_1\mathbf{n}_0]+\beta_4\left[\frac{\nabla T}{T}\,\mathbf{n}_0\right]+\beta_5\mathbf{A}_1+\beta_6\frac{\nabla T}{T}\right\}, \tag{52}$$

where

$$\mathbf{A}_1 = \frac{\nabla N_1}{N_1} + \frac{\nabla T}{T} + \frac{e}{kT}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_0\mathbf{H}]\right); \quad a = 1 + 2R^2 + \frac{25}{4}\left(\frac{\omega}{\nu}\right)_{el}^4 +$$

$$+ \left(\frac{\omega}{\nu}\right)_{el}^2 (28 + 2R^2 + 9.2R),$$

$$b = 4.5R^2 + 5.4R + 1; \quad \alpha_2 = \frac{15}{4a}\left\{1 + 2R - \frac{5}{2}\left(\frac{\omega}{\nu}\right)_{el}^2\right\} - \frac{5.1 + 10R}{b};$$

$$\alpha_1 = \frac{1}{a}\left(\frac{13}{4} + \frac{17\sqrt{2}R}{4} + \frac{25}{4}\left(\frac{\omega}{\nu}\right)_{el}^2 + 2R^2\right) - \frac{1}{b}(4.5R^2 + 13.35R + 5.4);$$

$$\alpha_4 = \frac{15}{4a}\left(\frac{\omega}{\nu}\right)_{el}\left(\frac{23}{4} + \sqrt{2}R\right),$$

$$\alpha_3 = \frac{1}{a}\left\{\frac{25}{4}\left(\frac{\omega}{\nu}\right)_{el}^3 + \left(\frac{\omega}{\nu}\right)_{el}\left(\frac{259}{16} + \frac{13}{2}\sqrt{2}R + 2R^2\right)\right\}; \qquad \alpha_5 = -(5.1 + 10R)/b;$$

$$\alpha_6 = -(4.5R^2 + 13.4R + 3.4)/b; \quad \beta_2 = -\frac{13.2 + 20R}{b} +$$

$$+ \frac{5}{2a}\left\{\frac{5}{2} + \frac{5}{2}\sqrt{2}R + \left(\frac{\omega}{\nu}\right)_{el}^2\left(\sqrt{2}R - \frac{1}{2}\right)\right\} + \frac{0.4\,(7.5Z^{-2} + \sqrt{2M/m}\,N_2/N_1)}{8MZ^2/25m + (\omega/\nu)_{el}^2};$$

$$\beta_1 = -\frac{1}{b}(4.5R^2 + 17.3R + 5.4) + \frac{1}{a}\left(\frac{19}{4} + \frac{23}{4}\sqrt{2}R + \frac{5}{2}\left(\frac{\omega}{\nu}\right)_{el}^2 + 2R^2\right);$$

$$\beta_3 = \frac{1}{a}\left\{\frac{25}{4}\left(\frac{\omega}{\nu}\right)_{el}^3 + \left(\frac{\omega}{\nu}\right)_{el}(24.8 + 8\sqrt{2}R + 2R^2)\right\};$$

$$\beta_4 = \frac{5}{2}\left(\frac{\omega}{\nu}\right)_{el}\frac{1}{a}\left\{13.4 + 1.5\sqrt{2}R + 2.5\left(\frac{\omega}{\nu}\right)_{el}^2\right\} - \left(\frac{\omega}{\nu}\right)_{el}\frac{N_2}{N_1 Z}\Big/\left(\frac{8Z^2M}{25\,m} + \left(\frac{\omega}{\nu}\right)_{el}^2\right);$$

$$\beta_5 = -\frac{1}{b}(4.5R^2 + 17.3R + 5.4); \quad \beta_6 = -\frac{1}{b}(13.2 + 20R).$$

6. Up to now we have found the quantities of interest in each concrete case as averaged out by a "local" distribution function. Space and time dependence enter only through the externally acting forces and the mean statistical characteristics, and are obtained by solving the generalized hydrodynamical equations (6a–c). The complete solution of the problem of plasma motion requires, in addition to the given initial (and boundary) conditions for j, Π_{ik}, $\tilde{v}$ and q through T, v_0, N, also the Maxwell equations for the electromagnetic field

$$\operatorname{div} E = 4\pi \sum e_s N_s; \quad \operatorname{curl} \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \qquad \operatorname{curl} \mathbf{H} - \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t} = \frac{4\pi}{c}(\mathbf{j} + \sum N_s e_s \mathbf{v}_0). \tag{53}$$

As an example, we shall consider a plasma in a strong magnetic field (stationary case with axial symmetry). It follows, in this case, from the equation of continuity that

$$v_{0r} = \tilde{v}_r = j_r = 0. \tag{54}$$

From (21)–(22), we have:

$$\frac{1}{N_1}\frac{dN_1}{dr} + \frac{1}{ZN_2}\frac{dN_2}{dr} + \left(\frac{1}{Z} - \frac{1}{2}\right)\frac{dT}{Tdr} = 0, \tag{55}$$

from which it follows that

$$N_1 N_2^{1/Z} = N_{01} N_{02}^{1/Z}\,(T/T_0)^{-(2+Z)/2Z}, \tag{56}$$

where N_0, T_0 are the initial values of the density and the temperature. If, in particular, the gas is almost quasi-neutral, we have*:

$$N_1 = N_2 Z = \text{const}\, T^{(Z-2)/2(1+Z)}.$$

We obtain next a generalized equation for the conservation of momentum in a magnetic field from the equation for the radial component of the momentum

$$H^2/8\pi + (N_1 + N_2)\,kT + \int eE\,(ZN_2 - N_1)\,dr = \text{const}. \qquad (57)$$

In this case the equation of conservation of energy, to a good approximation (including sources), takes the form

$$-\frac{1}{r}\frac{d}{dr}\left\{\frac{rN_2\,(kT)^2\nu_{el}}{m\omega_{el}^2}\left(\sqrt{2} + \frac{15}{2Z} + Z + \sqrt{\frac{2M}{m}}\right)\frac{dT}{dr}\right\} = S, \qquad (58)$$

where S is the heat source.

Including the equations of the electromagnetic field and the equation of motion in the θ-direction, we have 5 equations for the 5 quantities N_1, N_2, E, T, and H, and accordingly the given boundary conditions completely determine the solution.

In conclusion, the author wishes to express his profound gratitude to Academician I. E. Tamm and to Prof. V. L. Ginzburg for valuable advice and discussion of the results.

[1] E. S. Fradkin, Otchet (Report) FIAN, 1950-1951.

[2] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases*, Cambridge (1939).

[3] R. Landshoff, Phys. Rev. 76, 904 (1949).

Translated by M. A. Melkanoff
238

*If, in particular, the plasma consists primarily of electrons and ions of $Z = 1$ amongst a mixture of ions of various types, one may obtain the following law for the density distribution: $N_{el} = N_{Z=1} = \text{const}\, T^{-1/4}$; $N_{Z_1} = \text{const}\, T^{-[(M_Z(3Z+4)-M_1(3Z+2)]/4(M_1+M_Z)}$, where M_Z is the mass of an ion of charge Z.