	$r_0^3\bar{V}_R 10^{38}$ Mev. см^3	$\bar{V}_R$, Mev.
Li	2,43	15
Be	1,77	10,3
B^{10}	1,55	9
B^{11}	2,13	12,3
C	1,37	7,9
N	0,99	5,8
O	1,37	7,9
F	1,00	5,8

In this fashion, the potential varies irregularly with the variation of the atomic number A. One does not succeed in explaining such a behavior by the variation of r_0 (it is true that the available data[6] on light nuclei are incomplete and preliminary).

It remains to note that the formula of Deser *et al.* follows from (1) if the potential V is optical and is expressed in terms of the scattering lengths in the following way[7]

$$\left.\begin{aligned} V(r) &= V_0\rho(r), \\ V_0 &= -\frac{3}{2}\left(\frac{\hbar}{\mu c}\right)^2 \mu c^2 \frac{1}{Ar_0^3}\left[\frac{3N+Z}{3}a_3 + \frac{2Z}{3}a_1\right], \end{aligned}\right\} \quad (2)$$

where $\rho(r)$ is the nucleon density in the nucleus, N the number of neutrons, a_1, a_2 the scattering lengths of the meson on the nucleus in the spin states ½ and ³⁄₂. Finally, Eq. (1) yields the same expression as Brueckner's for the imaginary part of the shift (half width of the level).

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Possibilities of Focussing in a Linear Accelerator With the Aid of Traveling Waves

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As is well known, a simultaneous radial and phase stability of the motion of the accelerated particle in a linear accelerator with a traveling wave can be obtained by introducing a periodic unhomogeneity in the wave-guide. This method of alternating-phase focussing was proposed independently by Ia. B. Fainberg[1], by Mullet[2] and by Myron and Good[3] in 1953. In the present note, a modification of this method is proposed: the simultaneous radial and phase stability of heavy particles is obtained by the action of the additional generator on the field of the focussing traveling wave.

In the case considered here, the non-relativistic equations of motion of the accelerated particle have the form

$$\begin{aligned} m\ddot{z} &= eE\left\{I_0(kr)\cos\left(\omega t - \omega\int\frac{dz}{v}\right) + \vartheta I_0(k_1r)\cos\left(\omega_1 t - \omega_1\int\frac{dz}{v_1}\right)\right\}, \\ m\ddot{r} &= -eE\left\{I_1(kr)\sin\left(\omega t - \omega\int\frac{dz}{v}\right) + \vartheta I_1(k_1r)\sin\left(\omega_1 t - \omega_1\int\frac{dz}{v_1}\right)\right\}, \end{aligned} \quad (1)$$

where the first term is determined by the action of the accelerating travelling wave, and the second by the focussing wave. The phase velocities of the accelerating and focussing waves are determined by the formulas

$$v = \sqrt{(2eE/m)\cos\varphi_s(z+z_0)},$$

$$v_1 = \sqrt{(2eE/m)\cos\varphi_s(z+z_0+g)}, \quad z_0 = mv_0^2/2eE\cos\varphi_s;$$

where v_0 is the calculated velocity of the injected particles.

Let us use the notation

$$\vartheta = E_1/E, \quad N = \omega_1/\omega, \quad k = \omega/v, \quad k_1 = \omega_1/v_1,$$

$$\lambda = 2\pi c/\omega, \quad \Delta z = z - z_s, \quad \beta = v/c,$$

$$\mu = \beta^3\lambda mc^2/\pi(Ng\cos\varphi_s)^2 eE, \quad \Phi = k[(\Delta z)_{\max} - (\Delta z)_{\min}].$$

We will consider as small the deviations of the non-synchronous particle from the corresponding synchronous values, as well as the magnitude of the synchronous phase:

$$|kr|, \ |k_1r|, \ |k\Delta z|, \ |k_1\Delta z|, \ |\varphi_s| \ll 1, \quad (2)$$

The ratio of the amplitudes of the focussing and accelerating waves and the difference in their phase velocities will also be considered as small:

$$\vartheta,\ N\vartheta,\ g,\ (v_1 - v)/v \ll 1. \qquad (3)$$

In the approximate formulas reported below, only the lowest-order terms are included.

In order to perform the integrations in (1), we start by solving the equation of motion of a synchronous particle by the method of successive approximations:

$$m\ddot{z}_s = eE\left\{\cos\left(\omega t - \omega\int\frac{dz_s}{v}\right) + \vartheta\cos\left(\omega_1 t - \omega_1\int\frac{dz_s}{v_1}\right)\right\},$$

$$m\ddot{r}_s = 0 \qquad (4)$$

with the use of the assumption that

$$(\vartheta/10N)(\beta\lambda/g)^2 \ll eE\lambda/mc^2\beta \ll 1. \qquad (5)$$

Using the obtained expression for $z_s = z_s(t)$ and Eq. (4), let us expand Eq. (1) in a series with respect to the deviations of the particle coordinates from the corresponding values of the synchronous particle coordinates. Let us average[4] (over a period of the high frequency perturbation) the linear equations of motion for the deviations. The averaged equations of motion have the form:

$$\ddot{\overline{\Delta z}} + \Omega_z^2\,\overline{\Delta z} = 0, \quad \ddot{\overline{r}} + \Omega_r^2\,\overline{r} = 0;$$

$$\Omega_z^2 = \frac{1}{2}\left(\frac{e\vartheta E}{mg}\right)^2 (t+t_0)^2 - \frac{\omega\varphi_s}{(t+t_0)\cos\varphi_s}, \quad \Omega_r^2 = \frac{1}{8}\left(\frac{e\vartheta E}{mg}\right)^2 (t+t_0)^2 + \frac{\omega\varphi_s}{2(t+t_0)\cos\varphi_s}; \qquad (6)$$

$$t_0 = mv_0/eE\cos\varphi_s.$$

If $\varphi_s = 0$, the solutions of (6) are

$$\overline{\Delta z} = C_1\sqrt{t+t_0}\,J_{1/4}\left\{\frac{e\vartheta E}{2\sqrt{2}\,mg}(t+t_0)^2\right\} + C_2\sqrt{t+t_0}\,J_{-1/4}\left\{\frac{e\vartheta E}{2\sqrt{2}\,mg}(t+t_0)^2\right\},$$

$$\overline{r} = C_1'\sqrt{t+t_0}\,J_{1/4}\left\{\frac{e\vartheta E}{4\sqrt{2}\,mg}(t+t_0)^2\right\} + C_2'\sqrt{t+t_0}\,J_{-1/4}\left\{\frac{e\vartheta E}{4\sqrt{2}\,mg}(t+t_0)^2\right\}.$$

On the other hand, if $\varphi_s \neq 0$, applying the WKB method, we get:

$$\overline{\Delta z} = \frac{C_3}{\sqrt{\Omega_z}}\cos\int_{C_4}^{t}\Omega_z(t')\,dt',$$

$$\overline{r} = \frac{C_3'}{\sqrt{\Omega_r}}\cos\int_{C_4'}^{t}\Omega_r(t')\,dt',$$

where $C_1, C_1', \ldots, C_4'$ are constants determined by the initial conditions. When $\Omega_z^2, \Omega_r^2 > 0$, the solution obtained for $\overline{\Delta z}$ and $\overline{r}$ are oscillating, which corresponds to a simultaneous radial and phase stability of the accelerated particle motion.

If one takes into account the non-linear terms in the equations of motion for the deviations, then one obtains, for the capture angle Φ and for the acceptable velocity spread $(\Delta\beta/\beta)_{acc}$ the expressions

$$\Phi = 3\frac{\mu(N\vartheta)^2 - \varphi_s}{\cos\varphi_s},$$

$$\left(\frac{\Delta\beta}{\beta}\right)_{acc} = \sqrt{\frac{2eE\lambda}{3\pi mc^2\beta}\frac{[\mu(N\vartheta)^2 - \varphi_s]^{3/2}}{\cos\varphi_s}},$$

which are accurate in the absence of resonance in the radial and phase oscillations[5,6].

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