



FIG. 2.

be neither the result of an excitation of Th atoms in the recoil process, nor the result of the excitation of the uranium atoms by the  $\alpha$ -particles going through the source; it is due only to the internal conversion on the  $L$ -shell of  $\text{Th}^{230}$ . In the experiment, the 53 keV  $\gamma$ -rays have been separated by using  $120 \mu$  of Sn as an absorber (thick line on Fig. 2).

The conversion coefficient has been determined from the ratio of the number  $N_R$  of Roentgen quanta (without absorber) to the number  $N_\gamma$  of 53 keV  $\gamma$ -quanta, normalized to the same number  $N_\alpha$  of recorded  $\alpha$ -particles.

The following result is obtained:  $N_R/N_\gamma = 130$ . The experimental error does not exceed 50%. The extrapolation of the theoretical data gives for the sum of the conversion coefficient on  $L_i$ ,  $L_{ii}$ , and  $L_{iii}$  shells, depending on the type of transition:

$E1$	$E2$	$E3$	$M1$	$M2$
$< 1,0$	$170$	$> 5 \cdot 10^3$	$\sim 25$	$> 500$

The comparison with the experimental result enables one to conclude that the observed radiation is of the electric quadrupole type. The ground state moment of even-even nuclei being equal to zero, and its parity  $+$ , the total angular momentum of the first excited state of  $\text{Th}^{230}$  has to be equal to 2, parity  $+$ . The result obtained confirms experimentally the assumption of the rotational character of the level, according to the model of A. Bohr.

<sup>1</sup>L. A. Gold'in *et al.*, Academy of Science USSR Session on the Peaceful Use of Atomic Energy (Phys.-Math. Section) p. 226 (1955).

<sup>2</sup>T. Teillac, *Compt. rend.* **230**, 1056 (1950).

Translated by E. S. Troubetzkoy

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## Level Shift of $\pi$ -Mesonic Atoms

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(Submitted to JETP editor December 18, 1956)

*J. Exptl. Theoret. Phys. (U.S.S.R.)* **32**, 624 (March 1957)

IN recently completed experiments<sup>1-3</sup>, the energy levels of  $\pi$ -mesonic atoms of light elements have been measured. It was found out that the ground state level does not coincide with the energy level obtained by solving the Schrödinger equation in the potential of a point charge, taking into account the corrections for the finite size of the nucleus and the vacuum polarization. Theoretically, this problem has been investigated by Deser *et al.*<sup>4</sup> who obtained an equation expressing the level shift of  $\pi$ -mesonic atoms of light elements in terms of the scattering length of the  $\pi$ -meson on the nucleon; the assumption is made, however, that the scattering amplitude of the  $\pi$ -meson on the nucleus is, for small energies, equal to the sum of the scattering amplitudes on individual nucleons. An attempt to take into account the binding of the nucleons in the nucleus was made by Brueckner<sup>5</sup>, but the agreement with experiment is worse than for the results of Deser *et al.*

We will describe the non-electromagnetic interaction between the  $\pi$ -meson and nucleus by a complex  $V$ , different from zero in a region of the order of the nuclear dimensions. For a sufficiently light nucleus, we apply the perturbation theory, and obtain the following expression for the shift of the ground state level of the  $\pi$ -mesonic atom:

$$\Delta E = \int \psi_0^* V \psi_0 d\tau \cong \frac{4}{3} (r_0 / a)^3 \bar{V} Z^3 A, \quad (1)$$

where  $A$  and  $Z$  are the atomic weight and charge of the nucleus;  $\bar{V}$  is the mean value of the interaction potential;  $a = \hbar^2 / \mu e^2$ ,  $\mu$  is the meson mass;  $r_0 = R/A^{1/3}$ ,  $R$  being the nuclear radius. According to this formula, one can determine the product  $r_0^3 \bar{V}_R$  from the experimental data on the mesoatomic level shifts. If one takes  $r_0 = 1.2 \times 10^{-13}$  cm. then the following values of  $\bar{V}_R$  are obtained:

	$r_0^3 \bar{V}_R 10^{38} \text{Mev. cm}^3$	$\bar{V}_R, \text{Mev.}$
Li	2,43	15
Be	1,77	10,3
B <sup>10</sup>	1,55	9
B <sup>11</sup>	2,13	12,3
C	1,37	7,9
N	0,99	5,8
O	1,37	7,9
F	1,00	5,8

In this fashion, the potential varies irregularly with the variation of the atomic number  $A$ . One does not succeed in explaining such a behavior by the variation of  $r_0$  (it is true that the available data<sup>6</sup> on light nuclei are incomplete and preliminary).

It remains to note that the formula of Deser *et al.* follows from (1) if the potential  $V$  is optical and is expressed in terms of the scattering lengths in the following way<sup>7</sup>

$$\left. \begin{aligned} V(r) &= V_0 \rho(r), \\ V_0 &= -\frac{3}{2} \left( \frac{\hbar}{\mu c} \right)^2 \mu c^2 \frac{1}{A r_0^3} \left[ \frac{3N+Z}{3} a_3 + \frac{2Z}{3} a_1 \right], \end{aligned} \right\} (2)$$

where  $\rho(r)$  is the nucleon density in the nucleus,

$N$  the number of neutrons,  $a_1, a_2$  the scattering lengths of the meson on the nucleus in the spin states  $\frac{1}{2}$  and  $\frac{3}{2}$ . Finally, Eq. (1) yields the same expression as Brueckner's for the imaginary part of the shift (half width of the level).

The author expresses his gratitude to Ia. A. Smorodinskii for the position of the problem and for the discussion of the results.

<sup>1</sup> Proceedings of CERN Symposium 2, 417 (1956).

<sup>2</sup> M. Stearns and M. B. Stearns, Phys. Rev. 103, 1534 (1956).

<sup>3</sup> D. West and E. F. Bradley, Phil. Mag 8, 97 (1956).

<sup>4</sup> Deser, Goldberger, Baumann and Thirring, Phys. Rev. 96, 774 (1954).

<sup>5</sup> K. A. Brueckner, Phys. Rev. 98, 769 (1955).

<sup>6</sup> R. Hofstadter, Rev. Mod. Phys. 28, 214 (1956).

<sup>7</sup> Frank, Gammel and Watson, Phys. Rev. 101, 891 (1956).

Translated by E. S. Troubetzkoy  
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## Possibilities of Focussing in a Linear Accelerator With the Aid of Traveling Waves

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(Submitted to JETP editor December 20, 1956)

J. Exptl. Theoret. Phys. (U.S.S.R.) 32, 625-626

(March 1957)

As is well known, a simultaneous radial and phase stability of the motion of the accelerated particle in a linear accelerator with a traveling wave can

be obtained by introducing a periodic inhomogeneity in the wave-guide. This method of alternating-phase focussing was proposed independently by Ia. B. Fainberg<sup>1</sup>, by Mullet<sup>2</sup> and by Myron and Good<sup>3</sup> in 1953. In the present note, a modification of this method is proposed: the simultaneous radial and phase stability of heavy particles is obtained by the action of the additional generator on the field of the focussing traveling wave.

In the case considered here, the non-relativistic equations of motion of the accelerated particle have the form

$$\left. \begin{aligned} m\ddot{z} &= eE \left\{ I_0(kr) \cos \left( \omega t - \omega \int \frac{dz}{v} \right) + \vartheta I_0(k_1 r) \cos \left( \omega_1 t - \omega_1 \int \frac{dz}{v_1} \right) \right\}, \\ m\ddot{r} &= -eE \left\{ I_1(kr) \sin \left( \omega t - \omega \int \frac{dz}{v} \right) + \vartheta I_1(k_1 r) \sin \left( \omega_1 t - \omega_1 \int \frac{dz}{v_1} \right) \right\}, \end{aligned} \right\} (1)$$

where the first term is determined by the action of the accelerating travelling wave, and the second by the focussing wave. The phase velocities of the accelerating and focussing waves are determined by the formulas

$$v = \sqrt{(2eE/m) \cos \varphi_s (z + z_0)},$$

$$v_1 = \sqrt{(2eE/m) \cos \varphi_s (z + z_0 + g)}, \quad z_0 = mv_0^2 / 2eE \cos \varphi_s;$$

where  $v_0$  is the calculated velocity of the injected particles.

Let us use the notation

$$\left. \begin{aligned} \vartheta &= E_1 / E, \quad N = \omega_1 / \omega, \quad k = \omega / v, \quad k_1 = \omega_1 / v_1, \\ \lambda &= 2\pi c / \omega, \quad \Delta z = z - z_s, \quad \beta = v / c, \\ \mu &= \beta^3 \lambda m c^2 / \pi (Ng \cos \varphi_s)^2 eE, \quad \Phi = k [(\Delta z)_{\max} - (\Delta z)_{\min}]. \end{aligned} \right\}$$

We will consider as small the deviations of the non-synchronous particle from the corresponding synchronous values, as well as the magnitude of the synchronous phase:

$$|kr|, |k_1 r|, |k\Delta z|, |k_1 \Delta z|, |\varphi_s| \ll 1, \quad (2)$$