Radiation of a Point Charge Moving Along the Boundary between Two Media

V. E. PAFOMOV
P. N. Lebedev Physical Institute
Academy of Sciences, USSR
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WE here determine the angular distribution of radiated energy from an electron moving above the interface of two dielectrics. The special case of radiation by a point charge moving along the plane separation between a vacuum and dielectric has been considered by Danos[1] and Linhart[2], but Ref. 2 contains incorrect results and Ref. 1 contains misprints.

We assume that the electron is in uniform rectilinear motion with velocity v at distance d from the interface of two media with dielectric constants ϵ_1 and ϵ_2 which are assumed to be real. Let ϵ_1 be the dielectric constant of the medium in which the electron is moving. When the condition for Cerenkov radiation is satisfied only in the second medium ($\epsilon_1\beta^2 < 1$; $\epsilon_2\beta^2 > 1$), all of the energy is radiated into the second medium, and the intensity distribution along the generating lines of the Cerenkov cone is

$$\frac{dW}{dz} = \frac{2e^2}{\pi v^2} \int\limits_{\varepsilon_2\beta^2>1} \omega d\omega \int\limits_0^\pi d\varphi \frac{[(\varepsilon_2\beta^2-1)(\varepsilon_1+\varepsilon_2)\cos^2\varphi + \varepsilon_2(1-\varepsilon_1\beta^2)](\varepsilon_2\beta^2-1)\sin^2\varphi}{(\varepsilon_2-\varepsilon_1)[(\varepsilon_1+\varepsilon_2)\sin^2\varphi + \varepsilon_2\beta^2(\varepsilon_2\cos^2\varphi - \varepsilon_1\sin^2\varphi)]} \times \exp\left\{-2d\frac{\omega}{v}[(\varepsilon_2-\varepsilon_1)\beta^2 - (\varepsilon_2\beta^2-1)\sin^2\varphi]^{1/2}\right\}, \quad (1)$$

where φ is the azimuth whose zero is such that the plane $\varphi = \pi/2$ is perpendicular to the interface of the two media*. The Cerenkov cone is defined as in the homogeneous problem by the condition $n\beta\cos\vartheta = 1$, and since this condition is satisfied only below the interface the cone will be semicircular.

Ginzburg and Frank[3] (see also Ref. 4) have considered the radiation from an electron moving along the axis of a channel cut through a dielectric. For wavelengths shorter than the channel radius, the radiation energy decreases exponentially as the radius increases. This is also true qualitatively for the present case.

When $\epsilon_1\beta^2 > 1$ and $\epsilon_2\beta^2 > 1$, the result depends on the ratio of ϵ_1 and ϵ_2. When $\epsilon_1 > \epsilon_2$, the distribution of energy radiated into the second medium is

$$\frac{dW}{dz} = \frac{2e^2c^2}{\pi v^4} \int\limits_{\varepsilon_2\beta^2>1} \omega d\omega \int\limits_0^\pi \frac{A}{B^2} d\varphi;$$

$$A = \{(\varepsilon_1\beta^2-1) + [\sqrt{(\varepsilon_1\beta^2-1)-(\varepsilon_2\beta^2-1)\cos^2\varphi}\cdot\sin\varphi + \sqrt{\varepsilon_2\beta^2-1}\cdot\cos^2\varphi]^2(\varepsilon_2\beta^2-1)\}(\varepsilon_2\beta^2-1)\sin^2\varphi,$$

$$B = \varepsilon_1\sqrt{\varepsilon_2\beta^2-1}\sin\varphi + \varepsilon_2\sqrt{(\varepsilon_1\beta^2-1)-(\varepsilon_2\beta^2-1)\cos^2\varphi} \quad (2)$$

When for $\epsilon_1\beta^2 > 1$ and $\epsilon_2\beta^2 > 1$ we have $\epsilon_1 < \epsilon_2$, then in the region where $\cos^2\varphi < (\epsilon_1\beta^2-1)/(\epsilon_2\beta^2-1)$, the integrand in (2) must be replaced by the integrand in (1).

The flux into the first medium which results from interference is expressed by a more complicated formula that we shall not present here. We shall only mention that for $d = 0$, which means motion in the plane of the boundary, this flux is obtained from (1) and (2) when ϵ_1 and ϵ_2 are everywhere interchanged.

Since our case is not symmetrical with respect to the electron trajectory, a force arises which deflects the electron from its rectilinear motion; this is of interest in some cases. This effect will be the object of a separate investigation.

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* For $\epsilon_1 = 1$, Eq. (2) is not transformed into the corresponding formulas of Refs. 1 and 2, because of inaccuracies in these articles.

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[2] J. G. Linhart, J. Appl. Phys. 26, 527 (1955)
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[4] B. M. Bolotovskii, Dissertation, Physics Institute, Academy of Sciences, USSR, 1955

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