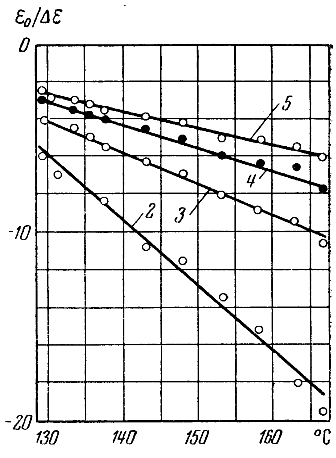

FIG. 3. 2, 3, 4, 5 —the same as in Fig. 2.

The comparison which has been made between the conclusions of the theory and the experimental data supports the possibility of applying the theory to polycrystalline $BaTiO_3$, at least in the paraelectric region. Regarding the ferroelectric region, satisfactory agreement between theory and experiment is observed only for temperatures no more than 10-12° below the Curie point.

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On Quantum Effects Occurring on Interaction of Electrons with High Frequency Fields in Resonant Cavities

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RECENTLY, a number of papers[1-7] were published in which an analysis was made of the quantum effects occurring during the interaction of electrons with high frequency fields in resonant cavities. And in the first place, the dispersion of the kinetic energy or the electron velocity, produced during their passage through the resonator, was calculated. Erroneous deductions are contained in some of the above mentioned papers, however, and in others[4,5,7], classical results were sometimes considered as quantum; quantum-mechanical methods were used for finding expressions, which are simpler, and which in the more general case can be obtained by a classical method. At the same time, the problem of the passage of electrons through resonators is one of the most important in electronics; moreover, the region of high frequencies ω and low temperatures T, when the condition of classicity $\hbar\omega \ll kT$ is disturbed, is acquiring increasingly greater interest. Therefore, we considered it appropriate to discuss briefly the problem which is stated in the title (this is carried out in greater detail in Ref. 8).

We will analyze classically the following problem: a non-relativistic electron enters into the resonator at the instant $t = 0$ with a kinetic energy $K_0 = mv_0^2/2$, and emerges from the resonator at the instant $t = \tau$ with a kinetic energy $K_\tau = mv_\tau^2/2$. Moreover, for simplicity we will consider the electric field E in the resonator along the electron path as homogeneous and directed along its velocity (such a case is completely real). When

$$E = E_1 \cos \omega t + (E_2 + E_0) \sin \omega t, \qquad \text{(a)}$$

we obtain:

$$m\frac{dv}{dt} = eE, \quad v_\tau = v_0 + \frac{e}{m\omega}[E_1 \sin \omega\tau + (E_2 + E_0)(1 - \cos \omega\tau)]. \qquad (1)$$

Let now E_1 and E_2 be random values, so that $\overline{E}_1 = \overline{E}_2 = 0$ and $\overline{E_1^2} = \overline{E_2^2} = \overline{V^2}/d^2$, where d is the path traversed by the electron (thickness of the resonator), and $\overline{V^2}$ is the mean square of the fluctuating voltage; the averaging, which is denoted by the bar, was carried out over the corresponding ensemble of identical systems. As in the papers cited above, we shall also consider the action of the field in the resonator on the electron motion as a small effect, in the strength of which we can limit ourselves to the terms of the e^2 order, and for the time of the electron flight through the resonator we shall assume $\tau = d/v_0$. Then, as can be easily seen,

$$\overline{(\Delta K_\tau)^2} = \overline{K_\tau^2} - [\overline{K_\tau}]^2 = \frac{4e^2 v_0^2}{\omega^2 d^2} \overline{V^2} \sin^2 \frac{\omega\tau}{2} \quad (2)$$

$$= e^2 \overline{V^2} \left[\frac{\sin(\omega\tau/2)}{\omega\tau/2} \right]^2$$

In addition, since $(\overline{\Delta v})^2 \ll v_0^2$, the dispersion of the velocity $\overline{(\Delta v_\tau)^2} = \overline{(\Delta K_\tau)^2} m^{-2} v_0^{-2}$. Under the conditions when $\omega t \ll 1$, $\overline{(\Delta K_\tau)^2} = e^2 \overline{V^2}$, that is, we obtain the expression (for the field constant with time) that is evident from the law of the conservation of energy. If oscillations of various frequencies are present in the resonator, then

$$\overline{(\Delta K_\tau)^2} = e^2 \int_0^\infty \overline{|V_\omega|^2} \left[\frac{\sin(\omega\tau/2)}{\omega\tau/2} \right]^2 d\omega, \quad (3)$$

$$\overline{V^2} = \int_0^\infty \overline{|V_\omega|^2} d\omega.$$

When only one natural oscillation is taken into account, a resonator is equivalent to a circuit consisting of (for example) a series connected resistance R, self-inductance L and capacity C, so that the impedance of the circuit when an emf $\epsilon = ZI$ is applied will be equal to $Z = R + i(\omega L - 1/\omega C)$. In this type of circuit, if we consider the condition of the thermal equilibrium, the spectral density of the fluctuating emf, in accordance with the Nyquist quantum theory[9,10], will be equal to

$$\overline{|\varepsilon_\omega|^2} = \frac{2}{\pi} R(\omega) \left\{ \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1} \right\}, \quad (4)$$

the density of the square of the current $|I_\omega|^2 = \overline{|\varepsilon_\omega|^2} / |Z(\omega)|^2$ and the density of the square of the voltage on the condenser

$$\overline{|V_\omega|^2} = \frac{\overline{|\varepsilon_\omega|^2}}{C^2 \omega^2 |Z(\omega)|^2} \quad (5)$$

$$= \frac{\overline{|\varepsilon_\omega|^2}}{R^2 C^2 \omega^2 + (LC\omega^2 - 1)^2}$$

Substituting (4) into (5) and then (5) into (3), we obtain the final expression for $\overline{(\Delta K)_\tau^2}$. For a weakly damped resonator with a frequency $\omega_0 = (LC)^{½}$, as can be easily shown, we will obtain from the general expression for $\overline{(\Delta K_\tau)^2}$ (see, for example, Ref. 10):

$$\overline{(\Delta K_\tau)^2} = \frac{e^2}{C(\omega_0)} \left(\frac{\hbar\omega_0}{2} + \frac{\hbar\omega_0}{e^{h\omega_0/kT} - 1} \right) \times \left(\frac{2}{\omega_0 \tau} \sin \frac{\omega_0 \tau}{2} \right)^2 \quad (6)$$

Senitzky[4] and Weber[7] obtained these results by using the quantum-mechanical theory of perturbations. The corresponding calculations are first, more complex and less clear, and second, are limited by the region of small damping. At the same time, these results hold for any damping of the circuit. A more detailed comparison of the obtained expressions with those given in Refs. 4,7 was carried out in Ref. 8. In the same paper, an analysis was made of the problem under discussion within the frame of canonical formalism, which is limited by the case of weak damping and, obviously, will lead to Eq. (6).

It should be noted also that in quantum-electrodynamic solution of the problem of the electron passage through a resonator, the same as in a number of other cases, the use of the ordinary theory of perturbations is limited (see Ref. 12 and also Refs. 8, 11), and in some cases it becomes necessary to use a more complex procedure of calculations[3, 12] which in the final count leads to essentially classical results.

From formulas (4) – (8) we see that the total quantum effect in the problem of the passage of electrons through a resonator is determined by the account of the quantum fluctuations of the radiation in the resonator and, in particular, of the zero oscillations with the energy $\hbar\omega/2$. In Refs. 4 and 5, it is asserted that there exists also a quantum effect associated with the account of the wave properties of electrons. It can be shown, however, that the dispersion of the electron velocity, as well as the dispersion of the field, calculated in Refs. 4 and 5, and associated with the scattering of the initial position and the momenta of the electrons, can be obtained also as a result of a considerably simpler classical calculation[8]. The single quantum element, moreover, consists in the fact that the initial distributions of the electron velocity and momenta are limited by the ratio of uncertainty. This limitation, however, is apparently completely insignificant in practice, even if we disregard the fact that its existence does not change the assertion of the possibility of purely classical analysis of the electron motion in a resonator within the accuracy with which the calculations were carried out in Refs. 4 and 5.

In conclusion, it should be noted that the portion of the velocity dispersion determined by the zero vibrations, which in the case of (6) is equal to

$$\overline{(\Delta v_\tau)^2} = e^2 \hbar\omega / 2Cm^2 v_0^2, \quad (b)$$

is quite small[8], although it may be of theoretical interest in the analysis of the performance of electronic instruments.

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On the Problem of K^0 Decays

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If we assume that in K-meson decays parity is conserved, then from the whole of the experimental data it apparently follows that there exist two mesons τ and θ with spin and parity 0^- and 0^+ respectively. Then it must be supposed that there exists a certain "degeneracy in parity" for the "strange particles"[1]. On the other hand one can assume that there exists only one K meson and that parity is not conserved in the decay interactions[2]. In the present note we point out one possibility for an experimental test of the hypothesis of nonconservation of parity.

We suppose that parity is conserved and consider the decay of a τ^0 meson. The possible decay schemes for it will be

$$\tau^0 \nearrow \pi^+ + \pi^- + \pi^0, \quad \tau^0 \searrow 3\pi^0, \qquad \tau^0 \to \begin{matrix}\mu^\pm \\ e^\pm\end{matrix} + \nu + \pi^\mp .$$

Like the θ^0 meson, the τ^0 meson must represent a mixture of charge-even and charge-odd components

$$\tau^0 = \left(\tau_s^0 + i\tau_a^0\right)/\sqrt{2} .$$

τ_s^0 will decay according to all four possible schemes, with the decay $\tau_s^0 \to 3\pi$ being the isotopic analogue of the τ^+ decay.

For τ_a^0 the decay $\tau^0 \to 3\pi^0$ is forbidden, and the decay $\tau^0 \to \pi^+ + \pi^- + \pi^0$ must go into states with orbital angular momentum different from zero and will be suppressed, so that the main decay for it will be

$$\tau_a^0 \to \begin{matrix}\mu^\pm \\ e^\pm\end{matrix} + \nu + \pi^\mp .$$

For both components the lifetime will be of the order of 10^{-7} sec.[3]

The situation is fundamentally changed if we assume that there exists one K meson but that decays occur with nonconservation of parity. In this case the main decay for the K_s^0 component will be $K^0 \to \pi^+ + \pi^-$; this decay is a fast one, so that the lifetime of K_s^0 will be $t \sim 10^{-10}$ sec. The charge-odd component, for which two-meson decay is impossible[4], will decay mainly according to the schemes

$$K^0 \to \begin{matrix}\mu^\pm \\ e^\pm\end{matrix} + \nu + \pi^\mp \quad \text{or} \quad K^0 \to 2\pi + \gamma$$

with lifetime $t \sim 10^{-8}$-10^{-7} sec.

Let us consider the decay curve of $\tau^0 \to \pi^+ + \pi^- + \pi^0$. In the case of conservation of parity, we must observe two slightly separated exponentials with nearly equal lifetimes $t \sim 10^{-7}$ sec. But in the case of nonconservation of parity we must observe together with an exponential of lifetime $t \sim 10^{-8}$-10^{-7} sec a short-lived component with lifetime of the order of 10^{-10} sec.

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