when $A = 200$; therefore, the first term can be neglected in proton transitions. For neutron transitions the second term disappears and the transition probability is determined entirely by the first term in the square brackets. It is interesting to compare the probability of and $E\,(L+1)$ transitions of neutrons:

$$W_{ML}/W_{E(L+1)} = S_{ML}/S_{E(L+1)}$$

$$(\mu_0 kA^L/2z)^2 \leqslant (S_{ML}/S_{E(L+1)})\;(2kA^{L-1}/1840)^2.$$

When $k \approx 10$ and $A = 100$ the probabilities $W_{M1}$ and $W_{E2}$ are of the same order of magnitude, and the radiation probabilities of the higher multipoles $M2$ and $M3$ are comparable even at low energies with the probabilities for $E3$ and $E4$ transitions.


[1] S. A. Moszkowski, Phys. Rev. 89, 474 (1953).

[2] J.H.D. Jensen and M. Goeppert-Mayer, Phys. Rev. 85, 1040 (1952).

[3] Berestetskii, Dolginov and Ter-Martirosian, J. Exptl. Theoret. Phys. (U.S.S.R.) 20, 527 (1950).


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## A Dispersion Relation for All Scattering Angles


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A DISPERSION relation for forward scattering has been obtained by Goldberger *et al.*[1,2]. In the present note a relation is obtained between the imaginary and real parts of the scattering amplitude for all angles.

To obtain this relation one can use Goldberger's method[1], in which it is simplest to employ the coordinate system in which the combined momentum of the nucleons (incident and scattered) is zero. However, we shall derive our dispersion relation here by using some results obtained by Nambu[3].

According to Nambu[3], the Feynman matrix element for the scattering of bosons (with momentum $k$ and charge index $\alpha$) by fermions (with momentum $p$ and additional quantum numbers $\lambda$) can be represented independently of the type of interaction in the form

$$F_{\alpha\beta}(k,\ \alpha;\ p,\ \lambda;\ k',\ \beta;\ p',\ \lambda') \tag{1}$$

$$= \bar{u}^{\lambda'}(p') \sum_{n=0,1} \{(\hat{k})^{n_1}(\tau_{\alpha\beta})^{n_2}\rho_{n_1 n_2}((p-p')^2,\ (p+k)^2)$$
$$+ (-k)^{n_1}(\tau_{\beta\alpha})^{n_2}\rho_{n_1 n_2}((p'-p)^2,$$
$$(p-k')^2)\}\ u^{\lambda}(p)\ \delta(p+k-p'-k'),$$

where

$$\rho(p^2, k^2) = \int \nu_{n_1 n_2}(p^2, u)\ \delta_+(k^2+u)\,du, \tag{2}$$

$$k'^2 = k^2 = -\mu^2, \quad \tau_{\alpha\beta} = \frac{1}{2}[\tau_\alpha, \tau_\beta],$$

in which the values of $u$ are given by $u = m^2$ and $u \geq (m+\mu)^2$.

The type of interaction affects only the dependence of $\nu$ on the arguments. By dividing $\delta_+(k^2)$ into $i\pi\delta(k^2)$ and $Pk^{-2}$ we obtain the imaginary and real parts, respectively, of the scattering amplitude $F_{\alpha\beta}$. It is convenient to continue our investigation in the coordinate system where the combined momentum of the nucleons is zero (i.e., $\mathbf{p}' + \mathbf{p} = 0$). Taking the $z$ axis in the direction of the vector $\mathbf{p}$ we have

$$p\,(E = \sqrt{m^2+p^2},\ 0,\ 0,\ p);\quad p'\,(E,\ 0,\ 0-p);$$
$$k\,(k_0 = \omega$$
$$= \sqrt{p^2+k_x^2+k_y^2+\mu^2};\ k_x,\ k_y,\ -p);\quad k'\,(\omega,\ k_x,\ k_y,\ p).$$

From (1) and (2) we obtain (hereafter $\delta(p+k-p'-k')$ will be omitted]:

$$F_{\alpha\beta} = D_{\alpha\beta} + i\tilde{A}_{\alpha\beta};\quad \tilde{A}_{\alpha\beta} = a_{\alpha\beta} + b_{\alpha\beta}; \tag{3}$$

$$D_{\alpha\beta}(k,\ p,\ \lambda'\lambda) = \bar{u}(p')^{\lambda} \tag{4}$$
$$\sum_{n=0,1}\Big\{(\hat{k})^{n_1}(\tau_{\alpha\beta})^{n_2}\int\frac{\nu_{n_1 n_2}((p-p')^2,\ u)\,du}{(p+k)^2+u}$$
$$+ (-k)^{n_1}(\tau_{\beta\alpha})^{n_2}\int\frac{\nu_{n_1 n_2}((p-p')^2,\ u)}{(p'-k)^2+u}\,du\Big\}\ u^{\lambda}(p')^{\lambda};$$

$$a_{\alpha\beta} = \pi\bar{u}^{\lambda'}(p') \tag{5}$$
$$\times \sum_{n=1,0}\{(\hat{k})^{n_1}(\tau_{\alpha\beta})^{n_2}\nu_{n_1 n_2}((p'-p)^2,\ -(p+k)^2)\}\ u^{\lambda}(p);$$

when $-(p+k)^2 \geqslant (m+\mu)^2$;

and $b_{\alpha\beta} = \pi \bar{u}^{\lambda'}(p')$ (6)

$\times \sum \{(-\hat{k})^{n_1} (\tau_{\beta\alpha})^{n_2} \nu_{n_1 n_2}((p'-p)^2, -(p'-k)^2)\} u^{\lambda}(p),$

when $-(p'-k)^2 \geq (m+\mu)^2$ or $m^2$. It is easily seen from Eqs. (1), (2) and (2a) that when the polarization of the nucleon does not change, the following dispersion relations hold *:

$$D^{(1)}_{\alpha\beta}(\mathbf{p}, \omega, \lambda', \lambda) - D^{(1)}_{\alpha\beta}(\mathbf{p}, 0, \lambda', \lambda) = \frac{\omega^2 - \omega_0^2}{\pi} \int_0^\infty \frac{2\omega' A^{(1)}_{\alpha\beta}(\mathbf{p}, \omega', \lambda', \lambda) d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)}; \quad (7)$$

$$D^{(2)}_{\alpha\beta}(\mathbf{p}, \omega, \lambda', \lambda) - \frac{\omega}{\omega_0} D^{(2)}_{\alpha\beta}(\mathbf{p}, \omega_0, \lambda', \lambda) = \frac{2\omega(\omega^2 - \omega_0^2)}{\pi} \int_0^\infty \frac{A^{(2)}(\mathbf{p}, \omega', \lambda', \lambda', \lambda) d\omega'}{(\omega'^2 - \omega^2)(\omega'^2 - \omega_0^2)}, \quad (8)$$

where

$$2D^{(1),(2)}_{\alpha\beta} = D_{\alpha\beta} \pm D_{\beta\alpha}, \quad 2A^{(1),(2)}_{\alpha\beta} = A_{\alpha\beta} \pm A_{\beta\alpha}, \quad A_{\alpha\beta} = a_{\alpha\beta} - b_{\alpha\beta}. \quad (9)$$

Repeating the calculations in Ref. 2 we obtain

$$D_+(\omega, \mathbf{p}) - \frac{1}{2}\left(1 + \frac{\omega}{\omega_0}\right) D_+(\omega_0\ \ ) - \frac{1}{2}\left(1 - \frac{\omega}{\omega_0}\right) D_-(\omega_0, \mathbf{p}) = \frac{2(\omega^2 - \omega_0^2)}{\pi} \int_0^\infty \frac{\omega' d\omega'}{\omega'^2 - \omega_0^2} \times \left[\frac{A_+(\omega', \mathbf{p})}{\omega' - \omega} + \frac{A_-(\omega', \mathbf{p})}{\omega' + \omega}\right] = B; \quad (10)$$

$$D_-(\omega, \mathbf{p}) - \frac{1}{2}\left(1 + \frac{\omega}{\omega_0}\right) D_-(\omega_0) - \frac{1}{2}\left(1 - \frac{\omega}{\omega_0}\right) D_+(\omega_0, \mathbf{p}) = \frac{2(\omega^2 - \omega_0^2)}{\pi} \int_0^\infty \frac{d\omega'}{\omega'^2 - \omega_0^2} \times \left[\frac{A_-(\omega', \mathbf{p})}{\omega' - \omega} + \frac{A_+(\omega', \mathbf{p})}{\omega' + \omega}\right] = B_1, \quad (11)$$

where $D_\pm(A_\pm)$ is the real (imaginary) part of the scattering amplitude of a positive (or negative) meson by a proton. Since the imaginary part of the amplitude in the integrands is taken for a fixed nucleonic energy and variable meson energy $\omega$, there is a direct relation to experiment only for those $A(\omega')$ for which $\omega'$ is higher than the threshold energy $\omega' \geq \omega_0 = \sqrt{\mu^2 + p^2}$.

In the energy range from 0 to $\omega_0$, $A(\omega)$ contains contributions not only from the "neutron" state ($u = m^2$) but from the entire spectrum of states with energies from $|m\mu - p^2| / |m^2 + p^2|^{1/2}$ to $\omega_0$.

The contribution of the "neutron" state can be calculated exactly since it is equal to

$$-\pi(\bar{u}(p')\Gamma(p', k')u(p'-k')) \times (\bar{u}(p'-k')\Gamma(p'-k', p-k'-p)u(p)) \times \delta\left(\omega' - \frac{\mu^2 + 2p^2}{2(m^2+p^2)^{1/2}}\right); \quad (12)$$

here $\Gamma(p, s)$ is $g\gamma_5\tau$ when $p^2 = -m^2$, $s^2 = -\mu^2$ (where $g$ is the exact renormalized coupling constant in the pseudoscalar theory).

It is easily seen that $A(\omega')$ in the continous energy spectrum from $|m\mu - p^2|/(m^2 + p^2)^{1/2}$ to $\omega_0$ can be obtained by analytic continuation in the region of imaginary momenta $k$.

We have finally

$$B(k) = \frac{2k^2}{\pi} \int_0^\infty \frac{dk'}{k'} \times \left[\frac{A_+(k', p)}{(\mu^2 + p^2 + k'^2)^{1/2} - (\mu^2 + p^2 + k^2)^{1/2}} + \frac{A_-(k', p)}{(\mu^2 + p^2 + k'^2)^{1/2} + (\mu^2 + p^2 + k^2)^{1/2}}\right] + \frac{2k^2}{\pi} \int_0^\alpha \frac{dk'}{k'} \times \left[\frac{A_+(ik', p)}{(\mu^2 + p^2 + k^2)^{1/2} - (\mu^2 + p^2 - k'^2)^{1/2}} - \frac{A_-(k', p)}{(\mu^2 + p^2 - k'^2)^{1/2} + (\mu^2 + p^2 + k^2)^{1/2}}\right] + \frac{g^2(\mu^2 + 2p^2)(m^2 + p^2)^{1/2} k^2}{2(\mu^2 + p^2) m^3 \{\sqrt{\mu^2 + p^2 + k^2} + (\mu^2 + 2p^2)/2(m + p^2)^{1/2}\}}, \quad (13)$$

where

$$\alpha = p\left(\frac{2m\mu + \mu^2}{m^2 + p^2}\right)^{1/2};$$

$$B_1(k) = \frac{2k^2}{\pi}\int_0^\infty \frac{dk'}{k'(\mu^2+p^2+k'^2)^{1/2}}\left[\frac{A_-(k',p)}{(\mu^2+p^2+k'^2)^{1/2}-(\mu^2+p^2+k^2)^{1/2}} + \frac{A_+(k',p)}{(\mu^2+p^2+k'^2)^{1/2}+(\mu^2+k^2+p^2)^{1/2}}\right]$$
$$+\frac{2k^2}{\pi}\int_0^\alpha \frac{dk'}{k'(\mu^2+p^2-k'^2)^{1/2}}\left[\frac{A_-(ik',p)}{(\mu^2+p^2+k^2)^{1/2}-(\mu^2+p^2-k'^2)^{1/2}} - \frac{A_+(ik',p)}{(\mu^2+p^2-k'^2)^{1/2}+(\mu^2+p^2+k^2)^{1/2}}\right]$$
$$-\frac{g^2(\mu^2+2p^2)(m^2+p^2)^{1/2}k^2}{2m^3(\mu^2+p^2)\{(\mu^2+p^2+k^2)^{1/2}-(\mu^2+2p^2)/2(m^2+p^2)^{1/2}\}}. \qquad (14)$$

* After the present work was completed (January 1956) the author learned that B. L. Ioffe and A. Salam had also obtained the relation (8).

[1] M. L. Goldberger, Phys. Rev. 99, 979 (1955).

[2] Goldberger, Miyazawa, and Oehme, Phys. Rev. 99, 986 (1955).

[3] Y. Nambu, Phys. Rev. 100, 394 (1955).

[4] A. Salam, Nuovo Cimento 3, 424 (1956).

Translated by I. Emin
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## A New Impulse Technique for Ion Mass Measurements

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PRECISE measurement of the mass of isotopes makes it possible to determine the binding energy of nucleons in a nucleus. The existing method of measuring masses through the use of a mass spectrograph involving magnetic deflection is not of sufficient accuracy for medium and heavy nuclei. Consequently, there is justification for proposing a new method of measuring masses.

We are familiar with the existing technique[1-3] for mass analysis which is based on measurement of the time of flight of ions with given energy over a specified distance. However, the energy spread of the ions formed in the source sets a limit of the order of 100 to 200 on the resolving power of this type of instrument. In the following a new method of energy focusing is proposed which is applicable to this type of spectrometer and which can considerably increase its resolving power.

This mass spectrometer has the form of a drift tube which is bounded at both ends by retarding fields with a linear potential distribution. The entire instrument is placed in a weak longitudinal magnetic field. A bunch of ions from the pulsed source is injected into the tube with simultaneous switching-off of the retarding field. Having entered a "potential well' the bunched ions begin to move from one repeller to the other, but the gain in the time of flight by the faster ions over the slower ions with given $e/M$ will be balanced by loss of velocity in the retarding field. Ions with identical $e/M$ and different energies will be focused at a certain point in the drift tube*. After a sufficient number of cycles, when ions with close values of $e/M$ have been separated out, that is, their shift in time of arrival at the focal point is greater than the duration of the pulses, the latter are deflected and recorded. In view of the fact that bunches of ions of different masses will be oscillating in the tube the voltage which must be applied to the deflecting plates will be switched off only at the instant when ions of the masses to be measured are passing. The transit time of ions with mass $M$, charge $Ze$ and energy $U$ (in volts) in a drift of length $l$ is

$$t_0 = l\sqrt{M/2UZe}. \qquad (1)$$

The time of motion of the ions in a retarding electric field of constant strength $E$ until they are stopped is

## ERRATA TO VOLUME 4

| | reads | should read |
|---|---|---|
| P. 218, column 2, Eq. (10) | $\ldots\ \xi(\sqrt{3}+2)\,(2-\sqrt{3})$ | $\ldots\ \xi^{(\sqrt{2}+2)/(2-\sqrt{3})}\ldots$ |
| P. 219, column 1, Eq. (11) | $\ldots\ (t\,\xi)^{\sqrt{3/2}}\ldots$ | $\ldots\ (t\,\xi)]^{\sqrt{3/2}}\ldots$ |
| P. 219, column 1, Eq. (12) | $y^2 = \rho^{2/3}$ | $y^2 - \rho^{2/3} \gg 1$ |
| P. 223, column 1, Eq. (45) | $\ldots\ (E_0\mu^{3/4})^{\sqrt{3/4}}$ | $\ldots\ (E_0\,\mu^{3/4})^{\sqrt{3}/4}$ |
| P. 223, column 2, Eq. (46) | $\ldots\ \mu^{3\sqrt{3/4}}\ \ldots$ | $\ldots\ \mu^{3\sqrt{3/4}}\ \ldots$ |
| P. 225, column 1, 3 lines above Eq. (1.1) | transversality | cross section |
| P. 225, column 1, 3 lines above Eq. (1.2) | transversality | cross section |
| P. 256, column 1, Eq. (37) | $\ldots\ \frac{55\sqrt{3}}{48}\ \ldots$ | $\ldots\ \frac{55}{\sqrt{3}\,48}\ \ldots$ |
| P. 289, column 2, Eq. (2) | | $I = \sum_n \frac{1}{2n+1} A_n \sum_{\nu=-n}^{n} \frac{1}{1+i\omega\tau} Y_{n\nu}(n_1)\, Y_{n\nu}(n_2)$ |
| P. 377, column 1, last line | $\delta_{35} = \eta - 21 \times \eta^5$ | $\delta_{35} - 21\,\eta^5$ |
| P. 436–7 | Figures 2 and 3 should be exchanged. | |
| P. 449, column 1, last Eq. | $\ldots\ Y_{lm}\ \varphi\sigma\ \alpha$ | $\ldots\ Y_{lm}\ \varphi_\sigma\ \alpha$ |
| P. 449, column 2, Eq. (12) | $\ldots\ W(l, j, \sigma\ 1;\ j)\ldots$ | $\ldots\ W(l, j, \sigma\ 1; \sigma\ j)\ldots$ |
| P. 451, column 1, Eq. (7) | $\ldots\ D^{(1)}_{\alpha\ \beta}(p, 0, \lambda'\ \lambda) = \ldots$ | $\ldots\ D^{(1)}_{\alpha\ \beta}(p, \omega_0, \lambda', \lambda) = \ldots$ |
| P. 541, column 1, Eq. (28) | $M^{*\,\mathrm{monex}}_{++}$ | $M^{*\,\mathrm{monex}}_{+}$ |
| P. 543, column 2, Eq. (35) | $\ldots \int_{\rho^2 - \tau^2 + l_0^2}$ | $\ldots \int_{\rho^2 < \tau^2 + l_0^2} \ldots$ |