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Ferromagnetic Superconductors

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The properties of ferromagnetic superconductors are considered. It is shown that the presence of spontaneous magnetization complicates the detection of ferromagnetic superconductors. However, use of thin films or wires, under known conditions and with large samples of high coercive force, should materially increase the possibility of detection of superconductivity in ferromagnetic metals and alloys.

TO date, superconductivity has never been detected in a ferromagnetic metal or alloy. This circumstance would not ordinarily be surprising, since many nonferromagnetic metals also fail to exhibit superconductivity.

On the other hand, in the present state of the microscopic theory of superconductivity, there exists no basis for denying the possibiliity of the existence of superconducting ferromagnets. Furthermore, in favor of such a possibility stands the fact that superconductivity is fundamentally related to the outermost electrons, while ferromagnetism is connected with the more deeply seated electrons. Moreover, many superconductors and ferromagnets are distinguished, at least insofar as their atomic states are concerned, only by *d*- or even *f*-electrons. Thus, superconducting lanthanum and hafnium differ from ferromagnetic gadolinium only in the 4*f*- and 5*d*-electrons,* while the ferromagnetic Fe, Ni and Co differ from superconducting Ti and V only by the addition of several 3*d*-electrons. Thus the absence of superconducting ferromagnets is, in each case, not a self-evident factor and requires an explanation.

The purpose of the present research is to point out the almost complete impossibility in practice, under ordinary conditions, of observing superconductivity in any sort of ferromagnets. It is shown that the presence in ferromagnets of spontaneous magnetization M_0 causes a large sample, even in the absence of an external magnetic field, to possess an induction $B_0 = 4\pi M_0$. Furthermore, it is quite natural, as will be shown below, that, in equilibrium, the superconducting phases can exist in large ferromagnets only if $B_0 = 4\pi M_0 < H^0_{c1}(0)$, where $H^0_{c1}(0)$ is the critical field of the large sample at $T = 0$ and consideration is not given to the effect of magnetization. The value of B_0 for $T = 0$ for Fe, Co, Ni, and Gd is, respectively, 22,000, 18,500, 6400 and 24,800 G, while $H^0_{c1}(0)$ for different superconducting elements lies in the range from 2600 G for Nb to 28.8 G for Cd.

Thus the probability of finding the superconductivity of ferromagnets in ordinary measurements is as small as in the case of nonferromagnetic super-

*It is possible that there are even closer superconducting neighbors to gadolinium since the question of the superconductivity of the rare earths is unfortunately not completely resolved. Moreover, there are several other ferromagnetic rare earths besides gadolinium.[1]

conductors placed in an external field with a magnetization of several thousand oersteds. In other words, a large ferromagnet can under ordinary conditions undergo a transition to the superconducting state only when its spontaneous magnetization M_0 (for $T = 0$) is very small ($B_0 = 4\pi M_0 \lesssim 10^3$), and the critical field is very large ($\gtrsim 10^3$).

These observations do not mean, however, that superconductivity of ferromagnets for which $B_0 = 4\pi M_0 > H^0_{c1}$ (0) can never be observed. On the contrary, under definite conditions (for thin films and wires, and also, perhaps, for large samples with high coercive force) superconductivity of ferromagnets can be observed if such superconductivity is otherwise possible (i.e., in the absence of the above-mentioned masking magnetic effect associated with the presence of spontaneous magnetization M_0).

1. In weak magnetic fields, $H \ll H_{cr}$, where H_{cr} is the critical field, the behavior of nonmagnetic superconductors is well described by the equations*

$$\text{curl}\,\Lambda\,\mathbf{j}_s = -(1/c)\,\mathbf{H}, \tag{1}$$

$$\partial\Lambda\mathbf{j}_s/\partial t = \mathbf{E}. \tag{2}$$

Furthermore, in the static case, to which we limit ourselves below, we must consider the field equation

$$\text{curl}\,\mathbf{H} = (4\pi/c)\mathbf{j}_s. \tag{3}$$

We get from (1) and (3) in the usual way (we assume that Λ = const):

$$\Delta\mathbf{H} - \delta_0^{-2}\mathbf{H} = 0, \quad \Delta\mathbf{j}_s - \delta_0^{-2}\mathbf{j}_s = 0, \tag{4}$$

$$\delta_0^2 = c^2\Lambda/4\pi = mc^2/4\pi e^2 n_s.$$

Above, no distinction has been made between the field H and the induction B=H + 4π M. For para- and diamagnets, this is practically true, but it is otherwise in the case of ferromagnets. It is easy to see that for B$\neq$ H, Eq. (1) can be replaced by

$$\text{curl}\,\Lambda\mathbf{j}_s = -(1/c)\,\mathbf{B}. \tag{5}$$

This substitution can be based on a whole series of considerations, namely: Eq. (5) and not (1) is compatible in the general case with the field equations and with Eq. (2). The latter follows from the very existence of superconductivity and is therefore preserved even in the transition to the case of magnets* ; evidence of this is given both by quantum mechanical considerations and also by the simple fact that the induction B is the field which acts on the current and which also determines the density of the superconducting current.

We get from (3) and (5) that

$$\Delta\mathbf{B} - \delta_0^{-2}\mathbf{B} = -4\pi\,\text{curl}\,\text{curl}\,\mathbf{M}, \tag{6}$$
$$\Delta\mathbf{j}_s - \delta_0^{-2}\mathbf{j}_s = c\delta_0^{-2}\,\text{curl}\,\mathbf{M}.$$

We shall consider below that the ferromagnetic is "ideal", i.e.,

$$\mathbf{B} = \mu\mathbf{H} + 4\pi\mathbf{M}_0, \quad \mathbf{M} = \mathbf{M}_0 + \frac{\mu - 1}{4\pi}\mathbf{H}, \tag{7}$$

where M_0 is the spontaneous magnetization and μ does not depend on H. For liquid helium temperatures, we can assume that M_0 is independent of temperature; at saturation, which is the usual condition, we can also set $\mu = 1$. Making use of (7), we have (it is assumed that μ= const):

$$\Delta\mathbf{B} - \delta^{-2}\mathbf{B} = -4\pi\,\text{curl}\,\text{curl}\,\mathbf{M}_0,$$
$$\Delta\mathbf{j}_s - \delta^{-2}\mathbf{j}_s = (c/\mu\delta^2)\text{curl}\,\mathbf{M}_0,$$
$$\delta^2 = \delta_0^2/\mu. \tag{8}$$

It is understood that these formulas are suitable for para- and diamagnetic materials for $M_0 = 0$. It is clear from (8) that in regions where M_0 = const, the induction and the superconducting current are damped out within the penetration depth δ . On the boundaries of the domains, where curl $M_0 \neq 0$, there are sources of induction and of current. If the sample consists of a single domain (which is also assumed below if there is no

*We avoid here the problem of the validity of Eqs.(1) and (2), inasmuch as, in our opinion, there is no substantial reason for doubting them at the present time.[2]

*It follows from (2) and from the equation curl E = $-(1c)\,\partial\mathbf{B}/\partial t$ that

$$\frac{\partial}{\partial t}\left\{\text{curl}\Lambda\mathbf{j}_s + \frac{1}{c}\mathbf{B}\right\} = 0,$$

which is compatible with (5)

contrary limitation), then it is possible to set curl $M_0 = 0$ everywhere. On the boundary of the specimen, where M_0 has a discontinuity, we must use the boundary conditions

$$H_e = H_i, \; B_e = (B_i - 4\pi M_0)/\mu; \qquad (9)$$

the indices e and i refer respectively to the region outside the metal, where $\mu = 1$ and $M_0 = 0$, and to the region inside the metal at its boundary. In the general case, the tangential components of H, B and M_0 would seem to enter into Eq. (9), but this is not reflected in the results, since below, for concrete applications, the normal components will be considered equal to zero (we consider below only cylindrical samples that are parallel to the field).

It is clear from the foregoing that in the superconducting state in the absence of an external field ($H_e = 0$), $B = 0$ in the mass of a uniformly magnetized superconducting ferromagnet, while in the normal state in such a cylindrical sample, $H_i = 0$, $B_i = B_0 = 4\pi M_0$. The induction B_0 is created by the (molecular) surface current $i_{mol} = cM_0$, which flows over the surface of the sample. In the superconducting state, even in the absence of an external field near the surface of the metal, the superconducting current j_s flows in a layer of thickness of the order δ. This current screens the field of the current i_{mol} in the mass of the metal. In this case the equivalent superconducting surface current $i_s = \int_0^\infty j_s\,dz$ (the z axis is directed along the normal to the surface of the sample, which is located at $z = 0$) is determined from the condition (the quantities with index ∞ refer to the mass of the sample):

$$i_s = c(H_{i\infty} - H_e)/4\pi$$

$$= -cM_0/\mu = -cH_e/4\pi,$$

since $H_{i\infty} = -4\pi M_0/\mu$, inasmuch as $B_{i\infty} = \mu H_{i\infty} + 4\pi M_0 = 0$. For $H_e = 0$ and $\mu = 1$, $i_s = -i_{mol}$, i.e., the superconducting current accurately compensates the "molecular" current.

2. Let us go on to the problem of the destruction of superconductivity of large ferromagnets in a magnetic field (we call that sample "large" whose smallest dimensions are significantly larger than $\delta \sim 10^{-5}$ cm).

Here, as in the case of nonferromagnets, we must use only the fact that in the superconductor, $B = 0$ (we neglect the surface layer where $B \neq 0$). Moreover, it is necessary to make use of the expression for the density of magnetic energy of the ferromagnets:

$$w_m = \frac{1}{4\pi}\int_0^B \mathbf{H}\cdot d\mathbf{B} = \frac{\mu H^2}{8\pi} - \frac{2\pi M_0^2}{\mu} = \frac{B^2}{8\pi\mu} - \frac{\mathbf{M}_0\mathbf{B}}{\mu}, \qquad (10)$$

where Eq. (7) is assumed and the constant is so chosen that $w_m = 0$ for $B = 0$.

Considering a cylindrical sample with a cross section of arbitrary form, located in an external magnetic field H_0 parallel to its axis, we obtain the following expression for the free energy per unit volume of sample:

$$F_n = F_{n0} + \frac{\mu H_e^2}{8\pi} - \frac{2\pi M_0^2}{\mu} - MH_e \qquad (11)$$

$$= F_{n0} + \frac{2-\mu}{8\pi}H_e^2 - \frac{2\pi M_0^2}{\mu} - M_0 H_e,$$

where F_{n0} is the free energy after deduction of the energy of the magnetic field and the term $-MH_e = -M_0 H_e - [(\mu-1)/4\pi] H_e^2$ is the density of the energy of interaction of the sample with the external field.*

In the superconducting state the free energy will be the same as for nonferromagnets since in both cases

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*In the general case, the free energy (more accurately the corresponding thermodynamic potential) of a system in an external uniform field $\mathbf{H}_e$ has the form:

$$\int F\,dV = \int (F_0 + w_m)\,dV - \frac{1}{4\pi}\int (\mathbf{B} - \mathbf{H}_e)\,\mathbf{H}_e\,dV,$$

where the integral is carried out over all space and F_0 is the free energy for $B=0$. For the cylinder we must limit ourselves to integration over the volume of the specimen, after which we get for the energy density, referred to unit volume:

$$F = F_0 + (1/S)\int w_m dV - (H_e/4\pi S)\int (B - H_e)\,dV, \qquad (12)$$

where S is the cross section of the specimen and the integration is carried out over a section of unit length along the axis of the cylinder. In the normal state, when the field is uniform,

$$-\frac{H_e}{4\pi S}\int (B - H_e)\,dV = -\frac{H_e(B - H_e)}{4\pi} = -MH_e;$$

the latter expression has a simple meaning, well known from magnetostatics.

$$B = 0 \text{ and } -H_e(B - H_e)/4\pi = H_e^2/4\pi.$$

Thus,

$$F_s = F_{s0} + H_e^2/4\pi. \quad (13)$$

The critical field for a large sample is determined from the equality $F_n = F_s$, whence

$$H_{c1}(T) = \pm H_{c1}^0(T)/\sqrt{\mu} - 4\pi M_0/\mu, \quad (14)$$

$$H_{c1}^0 = \sqrt{8\pi(F_{n0} - F_{s0})}. \quad (15)$$

The field $H_{c1}^{(0)}$ is evidently the critical magnetic field in the case of a nonferromagnetic metal with the same difference $F_{n0} - F_{s0}$ as for the ferromagnet under consideration and with $\mu = 1$. The direction of the external field H_e will be considered positive always, by virtue of which $H_{c1} \geq 0$. As concerns the magnetization M_0, in the isotropic case considered, it can be directed either along the field ($M_0 > 0$) or against the field ($M_0 < 0$).

In the first case, which is in equilibrium, for $M_0 = |M_0| > 0$ we have

$$H_{c1}(T) = H_{c1}^{(0)}(T)/\sqrt{\mu} - 4\pi M_0/\mu \quad (16)$$

and superconductivity is possible only if

$$H_{c1}^{(0)}(0) > 4\pi M_0/\sqrt{\mu}. \quad (17)$$

The result (16) means that the critical field $H_{c1}^{(0)}\sqrt{\mu}$ for $M_0 = 0$ must be equal to $H_{c1}^{(0)} - H_i$ where $H_i = -4\pi M_0/\mu$ is the field for $B = 0$. For $\mu = 1$, it can also be shown that $H_{c1}^{(0)} = H_{c1} + B_0$ where $B_0 = 4\pi M_0$ is the induction in the absence of superconductivity. For a saturated specimen, $\mu = 1$ effectively, and the criterion goes over into the form originally introduced. As has already been shown, superconductivity can be observed only under extraordinary circumstances when the quantity $B_0 = 4\pi M_0$ is anomalously small*, and the field $H_{c1}^{(0)}(0)$ corresponding is sufficiently large, although it lies inside reasonable limits. A state diagram of case (16) is shown schematically in Fig. 1.

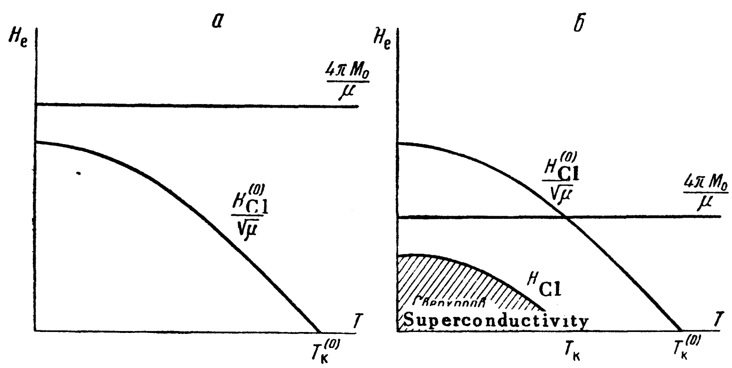

FIG. 1. $M_0 > 0$, $H_{C1}^{(0)}(0)/\sqrt{\mu} < 4\pi M_0/\mu$, superconductivity is not possible; (b) $M_0 > 0$, $H_{C1}^{(0)}/\sqrt{\mu} > 4\pi M_0/\mu$, $H_{C1}(T) = H_{C1}^{(0)}(T)/\sqrt{\mu} - 4\pi M_0/\mu$

For sufficiently clean and perfect materials, saturation takes place at very weak fields and the calculation just run through is valid (especially if anisotropy is not taken into account or if one does not consider the field directed along the axis of easy magnetization). In the absence of the field and for a multi-domain configuration, the criterion (17) remains suitable, inasmuch as the dimensions of the domains usually far exceed

*A sufficiently small value of M_0 can exist for alloys with concentration of components close to those for which ferromagnetism completely disappears (we assume that the transition from ferromagnetic alloys to nonferromagnetic is not a first order transition).

$\delta \sim 10^{-5}$ cm, and there must then be a current flow over the surface of the domain, canceling out the induction B in the interior of the domain. Moreover, upon appearance of superconductivity, the entire domain structure can, and in equilibrium must, change in the direction of enlargement or elimination of the domains, not to mention the fact that for long cylindrical samples, the role of the domain structure is generally relatively small.

We also note that for samples with large demagnetizing factor (for example, magnetization perpendicular to the plane of a thin disk) the field B in a sample in the nonsuperconducting state can be very small in comparison with $4\pi M_0$. Therefore the appearance of superconductivity will perhaps be facilitated in this case. However, it is also possible that edge effects will put at naught the advantage connected with reduced B. Thus the problem of the expediency of using samples with a large demagnetization factor demands special analysis which is yet to be carried out.

Let us now consider a uniformly magnetized cylindrical sample in which the magnetization $M_0 < 0$, i.e., it is directed against the field H_e. Such a state is metastable, but it is destroyed only by the achievement of a field H_c of value equal to the coercive force H_c which in some cases is of the order $4\pi M_0$, although it usually is significantly smaller than this value. In the case considered, there are, in accordance with Eq. (14), two critical field values:

$$H_{c1}^{+}(T) = \frac{4\pi|M_0|}{\mu} + \frac{H_{c1}^{(0)}(T)}{\sqrt{\mu}}, \quad (18)$$

$$H_{c1}^{-}(T) = \frac{4\pi|M_0|}{\mu} - \frac{H_{c1}^{(0)}(T)}{\sqrt{\mu}}.$$

It is easy to become convinced that the superconducting phase can exist only in a region of field H_e:

$$H_{c1}^{-}(T) \leqslant H_e \leqslant H_{c1}^{+}(T).^{*} \quad (19)$$

Here, if $H_{c1}^{-}(T)$ is negative in accord with Eq. (18), then this means that the critical field H_{c1}^{-} does not exist, and superconductivity will be observed in a zero field; a similar possibility exists upon fulfillment of condition (17). The region of existence of the superconducting phase in the case $M_0 < 0$ is clear from the schematic Fig. 2.

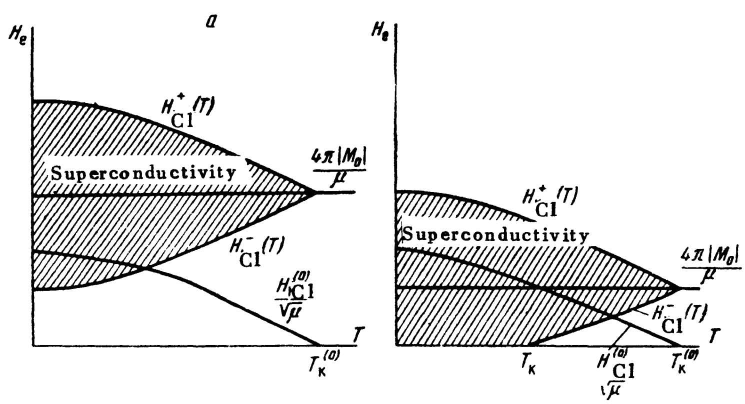

FIG. 2. (a) $M_0 < 0$, $H_{C1}^{(0)}/\sqrt{\mu} < 4\pi/M_0\, 1/\mu$; $H_{C1}^{+}(T) = 4\pi|M_0|/\mu + H_{C1}^{(0)}(T)/\sqrt{\mu}$, $H_{C1}^{-} = 4\pi|M_0|\mu - H_{C1}^{(0)}(5)/\sqrt{\mu}$; (b) $M_0 < 0$, $H_{C1}^{(0)}(0)/\sqrt{\mu} < 4\pi|M_0|/\mu$, $H_{C1}^{(H)}(T) = 4\pi|M_0|/\mu + H_{C1}^{(0)}(+)/\sqrt{\mu}$,

$H_{C1}^{-}(T) = 4\pi|M_0|/\mu - H_{C1}^{(0)}(T)/\sqrt{\mu} \gtreqless 0$.

It is obvious that this drawing can fully correspond to the actual case only if $H_{c1}^{+}(T) < H_c$. If the coercive force $H_c < H_{c1}^{-}(0)$ then superconductivity cannot in general be observed for $M_0 < 0$; if $H_{c1}^{-}(T) < H_c$ then upon increase in the field there ought to occur a transition to the superconducting state. The question as to in what

*We avoid the cases of supercooling or superheating, i.e., in region (19) the free energy of the superconducting phase F_s is less than or equal to the free energy of the normal phase F_n.

region this state will exist in the general case is difficult to answer with certainty inasmuch as we are dealing with a nonequilibrium system. From each case discussed it is clear that for use of a configuration with $M_0 < 0$, superconductivity is possible for the condition

$$H_{c1}^{(0)}(0) > (4\pi (M_0)/\sqrt{\mu}) - \sqrt{\mu} H_c, \quad (20)$$

which is weaker than (17).

In spite of this latter circumstance, the known values of M_0 and H_c for ferromagnetic elements do not permit us to hope for the discovery of superconductivity of large samples of them. In the case of certain alloys, utilization of configurations with $M_0 < 0$ can significantly increase the chances of discovery of superconductivity. However, greatest interest attaches to the use of ferromagnetic thin films (or wires), since in this case the critical field H_{cr}^0 increases strongly in comparison with H_{c1}^0, which allows us to hope for a substantial change in the situation. As will be shown, this hope is entirely justified.

3. Equation (1) is applicable only for a weak field $H \ll H_{cr}$ and therefore relates to Eq. (5) used in Sec. 1. On the other hand, in ferromagnetic superconductors, in contrast to nonferromagnetic ones, the field, generally speaking, is never weak everywhere. Therefore, in consideration of large samples, it is already necessary to extend to the ferromagnetic case the phenomenological theory of superconductivity developed in Ref. 3 and applied to arbitrary fields. However, in the case of large samples and in the absence of the intermediate state (and if the parameter $\varkappa \ll 1$; see Ref. 3), the theory of Ref. 3 leads to comparatively small departures from the theory of the Londons which is based on Eq. (1). From the point of view of Sec. 2, the chief consequence of Eq. (5) is that $B=0$ in the interior of the specimen. Inasmuch as this same conclusion also follows from the theory of Ref. 3 or, more precisely, from its generalization to ferromagnets carried out below, the results of Sec. 2 remain completely valid and the limited applicability of Eq. (5) is not important for us. In the transition to the problem of the destruction of superconductivity of specimens of small dimensions, the situation is completely different and one can start out only from the scheme put forth in Ref. 3.

In ferromagnetics, the fundamental equation for the free energy of the superconductor in the field (in the theory of Ref. 3), has, as is easy to see, the form

$$F_{sH} = F_{s0} + \frac{1}{2m}\left| -i\hbar\nabla\Psi - \frac{e}{c}\mathbf{A}\Psi \right|^2 + \frac{B^2}{8\pi\mu} - \frac{\mathbf{M}_0\mathbf{B}}{\mu}, \quad (21)$$

$$F_{s0} = F_{n0} + \alpha|\Psi|^2 + \frac{1}{2}\beta|\Psi|^4, \quad \mathbf{B} = \mathrm{curl}\,\mathbf{A}.$$

The entire difference of Eq. (21) from the corresponding equation in Ref. 3 consists of the use of Eq. (10) for the density of the magnetic energy, instead of the quantity $H^2/8\pi$ which is suitable for nonferromagnets; moreover, here curl A=B and not curl A=H. By virtue of the above, it is evident that for $M_0 = 0$ and $\mu=1$, Eq. (21) transforms into the expression for F_{sH} from Ref. 3. The equation for Ψ obtained from Eq. (21) has the same form as that in Ref. 3. Variation of Eq. (21) with respect to A under the condition div A=0 leads to the expression

$$\Delta\mathbf{A} = -\frac{4\pi}{c}(\mu\mathbf{j}_s + c\,\mathrm{curl}\,\mathbf{M}_0),$$

$$\mathbf{j}_s = (-ie\hbar/2m)(\Psi^*\nabla\Psi - \Psi^*\nabla\Psi) - (e^2/mc)\Psi^*\Psi\mathbf{A}. \quad (22)$$

This expression for A is in agreement with Eqs. (3) and (7) as it should be.

If Ψ =const, as we can assume in films[3,4], Eq. (22) takes the form:

$$\Delta\mathbf{A} - |\Psi_0|^2\delta^{-2}\mathbf{A} = -4\pi\,\mathrm{curl}\,\mathbf{M}_0,$$

$$\delta^2 = \delta_0^2/\mu = mc^2/4\pi e^2\mu|\Psi_\infty|^2,$$

$$|\Psi|^2 = |\Psi_0|^2|\Psi_\infty|^2, \quad (23)$$

where $|\Psi_\infty|^2 = -\alpha/\beta$ is the concentration of "superconducting electrons" (see Ref. 3) for $B=0$. At the same time $|\Psi|^2$ is the analogous concentration in the general case (for $B=0$, evidently, $|\Psi_0|^2 = 1$). Equation (23) is equivalent to Eq. (8) if we consider that $n_s = |\Psi|^2 = |\Psi_0|^2|\Psi_\alpha|^2$ and the values of δ in Eq. (8) and (23) coincide completely only for $|\Psi_0|^2 = 1$; inasmuch as for a large sample the latter equality does not take place in practice, we have not introduced different designations for the quantities δ in (8) and (23).

Let us now consider a film of thickness 2d, which is parallel to a field H_e. Taking the normal to the film along the z axis (at the center of the film, $z=0$), and the direction of the field H_e along the y axis, we see that the potential A and the current density $\mathbf{j}_s$ have components only along the x axis.

In this case, for $A_x = A$, we get

$$(d^2A / dz^2) - (\Psi_0^2 / \delta^2) A = 0, \quad B_y = B = dA / dz = \mu H + 4\pi M_0, \tag{24}$$

where the film is considered uniformly magnetized (therefore curl $M_0 = 0$; the field H and the magnetization M_0 are assumed also to be parallel to the y axis (like H_e).

It follows fromEq. (24) and the boundary conditions (9) that in the film

$$A = \frac{\delta B_i \,\mathrm{sh}\,(\Psi_0 z/\delta)}{\Psi_0 \,\mathrm{ch}\,(\Psi_0 d/\delta)}, \quad B = \frac{B_i \,\mathrm{ch}\,(\Psi_0 z/\delta)}{\mathrm{ch}\,(\Psi_0 d/\delta)}, \quad B_i = \mu H_e + 4\pi M_0 . \tag{25}$$

In the absence of an external field, $B_1 = B_0 = 4\pi M_0 \neq 0$, in contrast to nonferromagnets, in which, for $H_e = 0$, $B=0$, $A=0$ and $j_s = 0$. Therefore, in the latter case, for $H_e = 0$, the quantity $|\Psi_0|^2 = 1$. In ferromagnets, this is no longer so. The quantity Ψ_0 in the film, both for $H_e = 0$ and for $H_e \neq 0$, is determined from the requirement of a minimum of the free energy of the film, in which, for $H_e \neq 0$, one ought also to take into account the interaction of the magnetic moment of the film with the external field. In other words, one must minimize the free energy (12), i.e., in the case under consideration of a film with Ψ=const, the expression [see Ref. (21)]:

$$F_s = \frac{1}{2d} \int_{-d}^{d} \left\{ F_{sH} - H_e \left(\frac{B - H_e}{4\pi} \right) \right\} dz, \quad F_{sH} = F_{s0} + \frac{B^2}{8\pi\mu} - \frac{M_0 B}{\mu}. \tag{26}$$

Considering Eq. (25) and the fact that $H_{cl}^{(0)} = \sqrt{4\pi \alpha^2 / \beta}$ and $|\Psi|^2 = -(\alpha/\beta)|\Psi_0|^2$, we get*

$$F_s = F_{n0} + \frac{H_e^2}{4\pi} - \frac{B_i^2}{8\pi\mu} \frac{\mathrm{th}\,(\Psi_0 d/\delta)}{\Psi_0 d/\delta} + \frac{(H_{cl}^{(0)})^2}{8\pi} \cdot \Psi_0^2 (\Psi_0^2 - 2)$$

$$= F_n + \frac{B_i^2}{8\pi\mu} \left\{ 1 - \frac{\mathrm{th}\,(\Psi_0 d/\delta)}{\Psi_0 d/\delta} \right\} + \frac{(H_{cl}^{(0)})^2}{8\pi} \Psi_0^2 (\Psi_0^2 - 2) . \tag{27}$$

Here, the total free energy in the normal state (referred to unit volume), which is not different from Eq. (11), is equal to

$$F_n = F_{n0} + \frac{(2-\mu)}{8\pi} H_e^2 - \frac{2\pi M_0^2}{\mu} - M_0 H_e = F_{n0} - \frac{B_i^2}{8\pi\mu} + \frac{H_e^2}{4\pi} . \tag{28}$$

From the condition $\partial F_s / \partial \Psi_0 = 0$, we find the equation for Ψ_0 :

$$\left(\frac{B_i}{H_{cl}^{(0)}} \right)^2 = \frac{4\mu \Psi_0^2 (\Psi_0^2 - 1)\, \mathrm{ch}^2 (\Psi_0 d/\delta)}{1 - \mathrm{sh}\,(2\Psi_0 d/\delta)/(2\Psi_0 d/\delta)} . \tag{29}$$

For the critical value of the induction B_{cr} which is determined from the condition $F_s = F_n$, we get

$$\left(\frac{B_{cr}}{H_{cl}^{(0)}} \right)^2 = \frac{\mu \Psi_0^2 (2 - \Psi_0^2)}{1 - \mathrm{th}\,(\Psi_0 d/\delta)/(\Psi_0 d/\delta)} . \tag{30}$$

To find B_{cr} and Ψ_0 (for $B_i = B_{cr}$), we must solve Eqs. (29) and (30).

Equations (29) and (30) differ from those used in Refs. 3 and 4 only by the replacement of H_e and H_{cr} in Refs. 3,4 by $B_i / \sqrt{\mu}$ and $B_{cr} / \sqrt{\mu}$. Moreover, the quantity δ in Eqs. (29) and (30) differs from the δ_0 in Refs. 3,4 by the factor $\mu^{-1/2}$. Therefore it is not necessary to re-investigate Eqs. (29) and (30), and we only recall that for $d < (5/2)\,\delta$ destruction of superconductivity leads to a transition of second order, in which $\Psi_0 = 0$ for the critical field. Here

$$B_{cr} / \sqrt{\mu} = \pm \sqrt{6}\, (\delta/d)\, H_{cl}^{(0)} \equiv \pm H_{cr}^{(0)} \tag{31}$$

where $H_{cr}^{(0)}$ is the critical field for a film with the same values of δ, d and $H_{cl}^{(0)}$ but with $M_0 = 0$ and $\mu=1$.

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*The quantity $H_{cl}^{(0)}$ is the critical field for a large specimen for $M_0 = 0$ and $\mu=1$ (see Ref. 3, where this field is denoted by H_{cl}). We note that the restriction employed in Ref. 3 , which is connected with the consideration only of the region close to T_k , is not essential and it can be discarded.[5]

If $d > (\sqrt{5}/2)\,\delta$, a transition of first order takes place, in which again $B_{cr}/\sqrt{\mu} = \pm H_{cr}^{(0)}$. Inasmuch as $B_{cr} = \mu H_{cr} + 4\pi M_0$, where H_{cr} is the critical value of the external field H_e, we get

$$M_0 > 0: H_{cr}(T) = H_{cr}^{(0)}(T)/\sqrt{\mu} - 4\pi M_0/\mu, \quad (32)$$

$$M_0 < 0: H_{cr}^{\pm}(T) = 4\pi\,|M_0|/\mu \pm H_{cr}^{(0)}(T)/\sqrt{\mu}. \quad (33)$$

Thus precisely the same result is obtained as in the case of large samples, but withthe natural replacement of $H_{cl}^{(0)}(T)$ by $H_{cr}^{(0)}(T)$. Therefore there is no need of repeating what was done in Sec. 2. As concerns Figs. 1 and 2, they, being schematic also remain in force, inasmuch as the differences in the temperature dependence of $H_{cr}^{(0)}$ on $H_{cl}^{(0)}$ in this scheme is not essential.* However, in the quantitative relation, the situation changes sharply, for example, for $H_{cl}^{(0)}\,0 = 400$ Oe $\delta(0) = 5\times10^{-6}$ and $d = 2.5\times10^{-7}$ (the thickness of the film $2d = 50A$), $H_{cr}^{(0)}(0) \approx 20{,}000$ Oe which is already greater than $4\pi M_0$ for large samples of Ni and Co. For very thin films, the value of $4\pi M_0$ must decrease and furthermore, the possibility is not excluded of obtaining superconducting films of 2–3 times greater thinness than those used in the example treated.[6] Further, thin films must be uniform** and possess a relative large coercive force.[7] Therefore use of the configuration with $M_0 < 0$ [see Eq. (33)] can lead to a significant increase in the probability of discovering the existence of superconductivity. Finally, investigation of various ferromagnetic alloys especially with relatively small values of M_0, opens up additional possibilities, in particular if we consider that, for films, by virtue of their lesser density and also because of the small thickness, the value of M_0 should be smaller than for large pieces of metal.

If superconductivity will be discovered in very thin ferromagnetic films, then for further increase in the thickness of the film (for example, as a result of repeated saturation in the metal) the superconductivity decreases, as only $H_{cr}^{(0)}(T)$ remains smaller than $4\pi M_0$ (we are dealing with the case $M_0 > 0$, the value of μ is set equal to unity, as one should expect).

Experiments to show the superconductivity of ferromagnets deserved, we think, a great deal of attention, although it is possible that they will not succeed.

*Usually it is a satisfactory approximation to assume that

$$H_{Cl}^{(0)}(T) = H_{Cl}^{(0)}(0)\{1-(T/T_{Cr})^2\},$$
$$\delta(T) = \delta(0)\,[1-(T/T_{Cr})^4]^{-1},$$

whence the temperature dependence of $H_{cr}^{(0)}$ in the case (31) is also clear.

**By virtue of the polycrystalline composition of the films, it is perhaps expedient for obtaining sharp pictures to produce saturation of the film in a strong magnetic field.

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