$$\frac{1}{\beta}\frac{d\beta}{d\xi}=\frac{3g^2}{8\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi),\qquad(10)$$
$$\frac{1}{\alpha}\frac{d\alpha}{d\xi}=\frac{g^2}{4\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi),$$
$$\frac{1}{d}\frac{dd(\xi)}{d\xi}=-\frac{g^2}{\pi}\alpha^2(\xi)\beta^2(\xi)d(\xi).$$

The boundary conditions for β, α, d are equal to logarithmic accuracy, namely $\alpha(0)=\beta(0)=d(0)=1$ and it follows from Eq. (10) that:

$$\beta(\xi)=\alpha^{-3/2}(\xi),\qquad(11)$$
$$d(\xi)=\alpha^{-4}(\xi),$$
$$\alpha(\xi)=\left(1-\frac{5g^2}{4\pi}\xi\right)^{1/5}.$$

With the aid of Eq. (11) it is easy to find the relationship in this approximation between the primed charge $g_{\rm prim}$ and the renormalized charge. Actually, it is known that

$$g^2_{\rm prim}=\lim_{L\to\infty}(g^2\beta^2(L)\alpha^2(L)d(L))\qquad(12)$$
$$=\lim_{L\to\infty}\left[g^2\Big/\left\{1-\frac{5g^2}{4\pi}\ln\left(-\frac{L^2}{m^2}\right)\right\}\right]$$

or

$$g^2=\lim_{L\to\infty}\left[g^2_{\rm prim}\Big/\left\{1+\frac{5g^2_{\rm prim}}{4\pi}\ln\left(-\frac{L^2}{m^2}\right)\right\}\right].\qquad(13)$$

It is evident from Eqs. (12) and (13) that, at least in our approximation, no matter what kind the primed charge $g_{\rm prim}$ is, the experimental charge is equal to zero. This explains the fact that the solution of Eq. (11) at a finite g^2 changes sign at high momenta; moreover, a fictitious pole appears in $d(\xi)$, although, according to the formal general properties of the theory, $d(\xi)$ cannot become negative at large ξ.

One can easily become convinced that, if we substitute for g^2 its value which follows from theory in this approximation, then $d(\xi)$, as one would expect, does not change sign; however, the primed charge at $L\to\infty$ is completely shielded, and g^2 becomes equal to zero. The resultant difficulty, inherent in contemporary theories with point interaction, will be discussed in more detail in a separate paper.

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The Interaction of Extraordinary and Ordinary Waves in the Ionosphere and the Effect of Multiplication of Reflected Signals

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IT is known that the electromagnetic field of a wave traveling in an inhomogeneous magnetoactive medium (the ionosphere), generally speaking, cannot be represented by the superposition of independent extraordinary and ordinary waves. A consideration of the inhomogeneity of the medium leads to the conclusion that during the propagation of waves of one type in the medium, waves of another type appear. Strictly speaking, this interaction exists over the entire extent of the inhomogeneous medium; however, under ionospheric conditions (a slowly changing medium) the observable interaction appears only in limited regions, outside of which it is exceedingly slight. It is essential to speak of a division of the field into ordinary and extraordinary waves only under conditions of slight interaction. As a result of this it is possible to describe these waves in terms of geometrical optics; the interaction itself defines the very special nature of the field in regions of slight interaction separated by a region of considerable interaction.

With normal incidence of an electromagnetic wave upon a plane, laminated, ionized medium located in an external magnetic field, the strongest interaction between extraordinary and ordinary waves is observed during quasi-longitudinal propagation, when the angle between the direction of propagation and the direction of the external field is small. This interaction, which in the ionosphere produces the so-called multiplication of signals effect, can be explained in the following manner. The ordinary wave falling upon an inhomogeneous layer reaches the region where the index of refraction of the ordinary wave $n_1(z)$ and that of the extraordinary wave $n_2(z)$ are very close in value. In this region intense interaction of the two types of waves takes place; as a result of this interaction, an ordinary wave partly penetrates the region of imaginary values of $n_1(z)$ as an extraordinary wave; here the index of refraction of the latter $n_2(z)$ takes on real values and it is partly reflected as an ordinary wave. A wave of the second type passing through the region of interaction is reflected from a superincumbent region of zero values of the function $n_2(z)$ and on again passing through the region of interaction returns to the receiving point in the form of ordinary wave.

A calculation of this interaction in a nonabsorbing medium was first conducted by Ginzburg for two extreme cases: the first, when an ordinary wave passes almost entirely through the region of interaction (its coefficient of reflection R_1 is small); and the second, when $|R_1|$ is near unity and the coefficient of passage of the extraordinary wave D_2 is small (see reference 1, §79). The influence of absorption on the penetration effect is discussed in detail in the work of Rydbeck[2].

Application of the method of solution proposed in reference 3, which is devoted to the question of inelastic collisions between atoms, permits us to solve this problem fully to some approximation. The method indicated makes it possible to find an asymptotic representation of the particular solution which describes the real process of propagation of the waves in the medium at relatively great distances above and below the region of interaction. It then turns out that the ordinary wave falling upon a layer in the region of interaction produces a reflected wave of the very same type having a coefficient of reflection

$$|R_1| = 1 - e^{-2\delta_0}, \tag{1}$$

and its penetration of the region where $n_2^2 > 0$ characterized by the coefficient

$$|D_2| = e^{-\delta_0}. \tag{2}$$

In formulas (1), (2) the real value of δ_0 is defined by the integral

$$\delta_0 = -i\frac{\omega}{c}\oint \frac{n_2(z) - n_1(z)}{4}\,dz, \tag{3}$$

where the integral is taken around a closed contour encircling two particular points of the integrand at which $n_2 = n_1$; it is assumed that the ordinary wave decreases in the direction of positive z.

It should be noted that if the path of integration in Eq. (3) is taken along the line connecting the points where $n_2 = n_1$, Eq. (2) will give the expression for the coefficient of penetration obtained in reference 1 by an entirely different method, the applicability of which is limited to the case of small values of D_2 ($\delta_0 \gg 1$).

A study of Eqs. (2) and (3) shows that they are applicable also in another limiting case: where $|D_2| \approx 1$ and $|R_1|$ is small. With strong penetration ($\delta_0 \ll 1$) Eqs. (2) and (3) give

$$|R_1| \approx 2\delta_0;\ |D_2| \approx 1 - \delta_0, \tag{4}$$

and a calculation of the integral (3) under these conditions shows that these approximations of the coefficients of (4) fully coincide with the corresponding formulas obtained in reference 1 by a different method, in which the assumption of a small value for R_1 is used from the very beginning.

In our solution, moreover, we were able to compute the extraordinary wave arising as a result of the interaction. This wave is propagated in the direction of the pole of the function $n_2^2(z)$. The coefficient of reflection of this wave turns to be equal to

$$|R_2| = e^{-\delta_0}\sqrt{1 - e^{-2\delta_0}}. \tag{5}$$

As is easily verified, the computation of this coeffirient reduced to the satisfying of the relation

$$|R_1|^2 + |R_2|^2 + |D_2|^2 = 1.$$

In this manner the additonal absorption of the electromagnetic wave mentioned in reference 1 is connected with the appearance of the extraordinary wave which is then propagated in the direction of the sharp increase in its index of refraction and completely absorbed by the medium.

The phenomenon of complete absorption of this wave becomes more graphic if we consider the thermal movement of electrons. In reference 4, treating the kinetic energy of plasmic waves in an homogeneous plasma located in a magnetic field, it is shown that under these conditions the pole of the function $n_2^2(z)$ approaches infinity, although a sharp increase of the function itself is also maintained. Moreover, it turns out that in a consideration of the thermal movement of electrons the functions $n_1^2(z)$ and $n_2^2(z)$ are only negligibly distorted in the region of interaction. Consequently, the solution obtained without consideration of the thermal movement of electrons can be immediately extended to the more interesting case in which the possibility of the emergence of plasmic waves during interaction is considered. In this connection the extraordinary wave moving in the direction of the larger values of the function $n_2^2(z)$ acts as a slowly moving plasmic wave, the energy of which in a finite medium is expended in heating the plasma.

Finally, we note that, on the strength of the interaction, the reverse transformation of plasmic waves into electromagnetic waves, is also possible; this leads to the possibility of an emission of plasmic waves from an inhomogeneous magnetoactive medium in the form of electromagnetic radiation.

In conclusion I express my thanks to V. L. Ginzburg for suggesting the problem and for his assitance in the work.

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On Particle Energy Distribution at Multiple Formation

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IN Landau's work[1] there was developed a hydrodynamic theory of the formation of particles resulting from the collision of high energy nucleons. As is known, the resolution of this hydrodynamic problem concerning energy spread into evacuated space consists of two parts: wave motion and a nontrivial solution[2].

As regards the problem of multiple formation, the main role in the angular distribution of the particles is played by the nontrivial solution region, since it is here that the principal portion of the entropy of the system lies. The approximate solution of the problem of scattering, given by Landau, represented an asymptotic expression of the nontrivial solution of the scattering section remote from the boundary region separating it from the moving wave. In Landau's solution the latter was completely ignored. Accordingly, the question arises as to how far the disregard of the wave motion is justified when computing particle angular and energy distribution. It is to the examination of this problem that the present letter is devoted.

For the entropy S_p and energy E_p of the wave we have the expression:

$$S_p = nT_0^3 V_0 \int\limits_{x_1/l}^{ct/l} s_p u_p d\left(\frac{x}{l}\right), \qquad (1a)$$

$$E = mT_0^4 V_0 \int\limits_{x_1/l}^{ct/l} {}^4/_3 \varepsilon_p u_p^2 d\left(\frac{x}{l}\right). \qquad (1b)$$

Here l is the longitudinal extent of the system at the start of the scattering, $mT_0^4 4/3\, \epsilon_p u_p^2$ and $nT_0^3 s_p u_p$ are the densities of the energy and entropy of the wave, T_0 and V_0 are the initial temperature and volume of the system, n and m are constants. Integration is effected over the entire region occupied at the given moment by the moving wave, that is to say, from the boundary line it shares in common with the nontrivial solution section x_1 to the leading edge of the wave $x = ct$.

It should be noted that the coefficients before the integrals in (1a) and (1b) represent respectively the complete entropy and energy of the system. Therefore, the portions of the entropy α and the energy β contained in the moving wave will be equal to

$$\alpha = \frac{S_p}{S} = \int\limits_{x_1/l}^{ct/l} s_p u_p d\left(\frac{x}{l}\right), \qquad (2a)$$

$$\beta = \frac{E_p}{E} = {}^4/_3 \int\limits_{x_1/l}^{ct/l} \varepsilon_p u_p^2 d\left(\frac{x}{l}\right), \qquad (2b)$$

where u_p is a component of the four-velocity of the element.

Taking into consideration that $u_p \gg 1$ and making use of the Riemannian solution for a simple wave function, we can express ϵ_p, s_p and u_p as follows

$$\varepsilon_p = \left[\frac{(ct - x)(c - c_0)}{(ct + x)(c + c_0)}\right]^{2c_0/c}, \qquad (3a)$$

$$s_p = {}^4/_3 \left[\frac{ct - x}{ct + x}\,\frac{c - c_0}{c + c_0}\right]^{c/2c_0}, \qquad (3b)$$

$$u_p = {}^1/_2 \left[\frac{ct + x}{ct - x}\,\frac{c + c_0}{c - c_0}\right]^{1/_2}, \qquad (3c)$$

where c is the velocity of light and $c_0 = c/\sqrt{3}$ the velocity of sound. Substituting (3a), (3b) and (3c) in (2a) and (2b) and introducing a new variable $z = (ct - x)/l$, we obtain for α and β the following evident expressions:

$$\alpha = {}^1/_2 \left(\frac{c - c_0}{c + c_0}\right)^{1/_2[(c/c_0) - 1]} \times \int\limits_0^{z_1} \left(\frac{z}{((2ct/l) z)}\right)^{1/_2[(c/c_0) - 1]} dz, \qquad (4a)$$