quently, the number of shower particles observed was for all practical purposes the total number of shower particles that were produced in the layer of lead during the time of observation (523 hours).

Translated by V. A. Nedzel
25

SOVIET PHYSICS - JETP VOLUME 1, NUMBER 1 JULY, 1955

On the Theory of Energy Losses of Charged Particles Traversing a Ferromagnetic Material

D. IVANENKO AND V. N. TSYTOVICH
Moscow State University
(Submitted to JETP editor March 5, 1954)
J. Exper. Theoret. Phys. USSR 28, 291-296 (March, 1955)

An investigation of the effect of saturation of the energy losses of a charged particle passing through a ferromagnetic material is presented[1-3]. An analysis of the separation of the losses into ionization losses and Cerenkov radiation losses is also given.

THE energy losses of a charged particle passing through a ferromagnetic material has been investigated in a series of papers[1-3].

It is known[4,5] that when high velocity charged particles traverse a dielectric the energy losses approach saturation. These energy losses do not increase without limit as the energy of the charged particle increases but they are higher in materials which have a lower electron density. In addition to this, at higher velocities the losses by Cerenkov radiation play a special role[6-8].

For the case of a charged particle traversing a dielectric the analysis of the separation of the losses into ionization losses and Cerenkov radiation losses showed the essential dependence of this separation on the value of the damping coefficients in the dispersion formulas, which coefficients must not be assumed equal to zero[8,9].

The analogous situation can be expected to occur in the analysis of the separation of the losses into ionization losses and Cerenkov radiation losses when a particle traverses a ferromagnetic material.

[1] D. Ivanenko and B. C. Gurgenidze, Doklady Akad. Nauk SSSR 67, 997 (1949)

[2] D. Ivanenko and B. C. Gurgenidze, Vestn. Moscow State University 2, 69 (1950)

[3] Ch. Weizsäcker, Ann. Physik 17, 869 (1933)

[4] E. Fermi, Phys. Rev. 53, 485 (1940)

[5] N. Bohr, Atomic Particles Traversing Media (1950)

[6] P. A. Cherenkov, Doklady Akad. Nauk SSSR 2, 451 (1934)

[7] I. M. Frank and I. E. Tamm, Doklady Akad. Nauk SSSR 14, 107 (1937)

[8] M. Schönberg, Nuovo Cim. 8, 159 (1951)

[9] P. Budini, Nuovo Cim. 10, 236 (1953)

In addition, in a ferromagnetic material as in a dielectric, the question of this separation cannot be calculated correctly if the effect of damping is not considered finite[9].

When a high velocity charged particle traverses a substance, the loss of its energy depends essentially on the interaction between the atoms of the substance. In a ferromagnetic material the magnetic properties are determined, first, by an exchange interaction of the electrons of the substance (exchange energy) and secondly, by a magnetic interaction of the elementary magnetic moments (energy of magnetic anisotropy(see, for example, Vonsovskii and Shur[10]). In the following discussion, however, it will be shown the exchange interaction is not essential for the energy losses of charged particles which pass through a ferromagnetic material. A relatively small energy of magnetic isotropy, however, gives a contribution to the energy losses, but it is negligibly small in comparison to the ionization losses and Cerenkov radiation losses associated with the dielectric constant ϵ of the ferromagnetic material. Also included are some unexpected results of the analysis of the energy losses in a ferromagnetic material. We note also that the effect of saturation of the losses in a ferromagnetic material is analogous to the effect of saturation of the losses in a dielectric, where the losses in the ultra relativistic region depend only on the number of electrons, but not on the character of their coupling in the material.

If point charged particles, moving in a medium characterized by values of dielectric constant and

[10] S. V. Vonsovskii and Y. S. Shur, *Ferromagnetism*, Moscow (1948)

permeability ϵ and μ, have their kinematics denoted by $\mathbf{r}_\xi(t)$ and $\mathbf{v}_\xi(t)$ (position and velocity of the particle) then the potentials of the field obey the equations:

$$\nabla^2\mathbf{A} - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\mathbf{A}}{\partial t^2} = -\frac{4\pi\mu}{c}e\mathbf{v}_\xi\delta(\mathbf{r}-\mathbf{r}_\xi), \quad (1)$$

$$\nabla^2\phi - \frac{\varepsilon\mu}{c^2}\frac{\partial^2\phi}{\partial t^2} = -\frac{4\pi}{\varepsilon}e\delta(\mathbf{r}-\mathbf{r}_\xi). \quad (2)$$

Since we are interested in both ionization losses and Cerenkov radiation losses we will consider the values of ϵ and μ to be complex.

It is well known[11,12] that in calculating Green's function of the field equations for transparent media, a well-defined expression is not obtained because of the presence of poles on the real axis in the expression of the integrand. In order to circumvent these poles correctly we can use either retarded, advanced, or other types of functions[13,14]. In the cases we are considering, because of the complex values of ϵ and μ, the poles of the integrand of the Green's function will not lie on the real axis. The requirements for radiation will be fulfilled if the coefficients in the imaginary part of ϵ and μ are positive ($\phi = e^{-i\omega t}\phi_0$). The latter circumstance, corresponding to a dissipation of energy by moving elementary charges and currents, is dependent upon a given value of ϵ and μ. The condition of positive coefficients in the imaginary parts of ϵ and μ, where the damping goes to zero, leads directly to a retarded potential for the transparent media.

Thus for the Green's function of Eqs. (1) and (2) we obtain the expression:

$$G = \frac{1}{16\pi^4} \quad (3)$$

$$\int \frac{1}{k^2 - (\varepsilon\mu\omega^2/c^2)} \exp\{i\mathbf{kR} - i\omega T\}\, d\omega d^3k,$$

where $\mathbf{R} = \mathbf{r} - \mathbf{r}'$, $T = t - t'$. Hence, for the potentials ϕ and $\mathbf{A}$, we have

$$\phi = \frac{e}{4\pi^3}\int \frac{\exp\{i\mathbf{kR}_\xi - i\omega T\}}{\varepsilon(\omega)\,[k^2 - (\omega^2\varepsilon\mu/c^2)]} d\omega d^3k dt', \quad (4)$$

$$\mathbf{A} = \frac{e}{4\pi^3}\int \frac{\exp\{i\mathbf{kR}_\xi - i\omega T\}}{c\,[k^2 - (\omega^2\varepsilon\mu/c^2)]}\mu(\omega)\,\mathbf{v}_\xi d\omega d^3k dt'. \quad (5)$$

Here

$$\mathbf{R}_\xi = \mathbf{r} - \mathbf{r}_\xi(t').$$

The potentials in Eqs. (4) and (5) permit finding the energy losses for any moving charged particles $\mathbf{r}_\xi(t)$, $\mathbf{v}_\xi(t)$. With these we can develop, in particular, the theory dealing with the analysis of the energy losses of charged particles having relativistic motion in a magnetic field in certain transparent media (recombination phenomena of the electrons "luminescence" and "superlight", that is, Cerenkov radiation[12]).

In case of uniform motion along the z axis, we have

$$\phi = \frac{e}{\pi}\int \frac{d\varkappa}{\varepsilon}\exp\{i\varkappa(z - vt)\}K_0(\zeta r), \quad (6)$$

where

$$K_0(\zeta r) = \frac{\pi i}{2}H_0^{(1)}(i\zeta r), \quad \varkappa = \frac{\omega}{v}. \quad (7)$$

Here

$$\zeta = \varkappa\sqrt{1-\varepsilon\mu\beta^2}\ \ \mathrm{Sign\,Re}\,\varkappa\sqrt{1-\varepsilon\mu\beta^2}, \quad (8)$$

and $H_0^{(1)}$ is a cylindrical function.

For determining the energy losses due to collisions with the parameter of the collision greater than b, we calculate the flux of the Poynting vector through the surface of a cylinder of radius b, which surrounds the trajectory of the particle:

$$W_b = \frac{c}{4\pi v}\int_b [\mathbf{EH}]\,dS \quad (9)$$

$$= \frac{2e^2}{\pi v^2}\mathrm{Re}\int_0^\infty i\omega d\omega\left(\frac{1}{\varepsilon} - \mu\beta^2\right)\zeta^* b K_0(\zeta b)K_1(\zeta^* b).$$

Here $\mathbf{E}$ and $\mathbf{H}$ are the values of the electric and magnetic intensities created by the charged particle, $d\mathbf{S}$ is an element of surface of the cylinder. In the derivation of Eq. (9) we used the relations[11]

$$A_x = A_y = 0, \quad A_z = \varepsilon\mu\beta\phi;$$

$$E_z = \frac{\partial\phi}{\partial z}(\varepsilon\mu\beta^2 - 1), \quad E_x = -\frac{\partial\phi}{\partial x},$$

$$E_y = -\frac{\partial\phi}{\partial y}, \quad H_z = 0;$$

$$H_x = \varepsilon\beta\frac{\partial\phi}{\partial y}, \quad H_y = -\varepsilon\beta\frac{\partial\phi}{\partial x},$$

and used for ϕ the expression (6), as well as the formulas:

---

[11] D. Ivanenko and A. Sokolov, *Classical Theory of Fields*, Moscow, 2nd ed. (1951)

[12] V. N. Tsytovich, Vestn. Moscow State University 11 27 (1951)

[13] R. Feynman, Phys. Rev. 76, 749 (1949)

[14] Article edited by D. Ivanenko: "Most Recent Developments in Quantum Electrodynamics", Moscow (1954)

$$K_1(z) = -\frac{\partial}{\partial z} K_0(z); \quad \varepsilon(-\omega) = \varepsilon^*(\omega);$$

$$\mu(-\omega) = \mu^*(\omega).$$

In the following we shall consider the small value of the collision parameter which limits us in Eq. (9) to the first members of the series for the Bessel functions $K_0(\zeta b)$ and $K_1(\zeta^* b)$:

$$K_1(\zeta^* b) = \frac{1}{\zeta^* b} \quad K_0(\zeta b) = \frac{1}{2} \ln \frac{4}{3.17\zeta^2 b^2}. \tag{10}$$

The relations in Eq. (10) can be used if $|\zeta| b \ll 1$. For a ferromagnetic material, with $\epsilon = 1$ and μ determined by the dispersion formula[15] the generalized classical formula of Arkadév is used [see below, Eq. (19)]. This condition is given for ultrarelativistic motion by

$$\frac{\omega}{c} |\sqrt{1-\beta^2\mu_{\max}}| b \ll 1, \tag{11}$$

where μ_{max} is the maximum value of the magnetic permeability occurring in the region near the resonance frequency ω_0. If we neglect damping, then condition (11) is violated. Therefore in the following it is very essential to have a finite value of the damping. However, in the final result, after combining the ionization losses and the Cerenkov radiation losses, the damping can be neglected. The situation here is somewhat more complex than in the case of dielectrics[16], because the ranges of the integration with respect to frequency for Cerenkov radiation losses and for ionization losses partly overlap.

Thus, for a small range of the papameter we have

$$W_b = \frac{e^2}{\pi v^2} \operatorname{Re} \int_0^\infty i\omega d\omega \left(\frac{1}{\varepsilon} - \mu\beta^2\right) \ln \frac{4}{3.17\zeta^2 b^2}. \tag{12}$$

Assuming $\varepsilon = 1$, $\mu = \operatorname{Re}\mu + i \operatorname{Im}\mu$, $\gamma = \sqrt{3.17}$

$$\tan\phi = -\frac{\beta^2 \operatorname{Im}\mu}{1-\beta^2 \operatorname{Re}\mu}, \tag{13}$$

we have

$$W_b = \frac{2e^2}{\pi v^2} \int_0^\infty \omega d\omega \left\{ \operatorname{Im}\mu\beta^2 \ln \frac{2v}{\gamma\omega|\sqrt{1-\beta^2\mu}|\, b} + \frac{\phi}{2}(1-\operatorname{Re}\mu\beta^2) \right\}. \tag{14}$$

For small values of the damping coefficient, instead of Eq. (13), we can write

$$\phi = \frac{\beta^2 \operatorname{Im}\mu}{\beta^2 \operatorname{Re}\mu - 1} - \pi \quad \text{when } \beta^2 \operatorname{Re}\mu > 1; \tag{15}$$

$$\phi = -\frac{\beta^2 \operatorname{Im}\mu}{1-\beta^2 \operatorname{Re}\mu} \quad \text{when } \beta^2 \operatorname{Re}\mu < 1. \tag{16}$$

The relation (15) corresponds to the Cerenkov frequency, and Eq. (16) to the Bohr frequency.

Substituting Eqs. (15) and (16) in Eq. (14) we obtain the formulas for ionization and Cerenkov radiation losses in a ferromagnetic material with $\epsilon = 1$:

$$W^{\text{Cer}} = \frac{e^2}{v^2} \int_{\operatorname{Re}\mu\beta^2>1} (1-\operatorname{Re}\mu\beta^2)\,\omega d\omega, \tag{17}$$

$$W_b^{\text{Ion}} = \frac{e^2}{\pi c^2} \tag{18}$$

$$\int_0^\infty \omega d\omega \operatorname{Im}\mu \left\{ \ln \frac{4v^2}{3{,}17\omega^2 b^2 |1-\mu\beta^2|} - 1 \right\}.$$

In the following we will assume a concrete dispersion formula for the magnetic permeability*.

$$\mu = 1 + \frac{4\pi}{\beta} \frac{1-i\gamma x}{1-x^2-2i\gamma x}, \tag{19}$$

where

$$x = \frac{\omega}{\omega_0}, \quad \omega_0 = \frac{e}{mc}\beta I_s, \quad \gamma = \frac{e}{mc}\beta\varepsilon, \quad \beta = \frac{2k}{I_s^2},$$

I_s is the spontaneous magnetization, k is the coefficient of magnetic anisotropy, m is the mass of the electron.

We now stop to analyze the ionization losses [Eq. (18)]. Assuming the value of μ in the logarithmic part is equal to unity, we obtain the formula for the ionization losses, neglecting the polarization effect[9]. In the calculation of the ionization losses the value of μ is essentially the value near resonance[9]. Therefore we set

$$\mu = 1 + \frac{\mu_0 - 1}{1-x^2-2i\gamma x} \tag{20}$$

---

[15] L. D. Landau and E. M. Lifshits, Sow. Phys 8, 157 (1935)

[16] P. Kunin, Article in "Mesons", edited by I. Tamm, Moscow (1947)

* We emphasize that in Refs. 1 and 10 [e.g., see Ref. 1, D. Ivanenko and B. C. Gurgenidze, Doklady Akad. Nauk SSSR 67, 997 (1949) and Ref. 10, S. V. Vonsovskii and Y. S. Shur, *Ferromagnetism*, Moscow (1948)] there was a typographical error in the expression of the relationship between μ and frequency. We use this occasion to note that the paper of Sitenko [e.g., see Ref. 17, A. G. Sitenko, Zh. Tekhn. Fiz. 23, 2200 (1953)] , to which our attention was directed after the present paper was sent to the printer, is devoted to a closely related problem. However, the final results of the indicated paper do not agree with the reduced Eqs. (23) and (24), apparently as a result of some terms neglected in the calculations of Sitenko.

[17] A. G. Sitenko, Zh. Tekhn. Fiz. 23, 2200 (1953)

and we obtain the analogue of the Bloch formula[18,19] (see also Budini[9]) for the ionization losses in a ferromagnetic material with

$$W_b^{\text{Ion}} = \frac{e^2\omega_0^2(\mu_0 - 1)}{2c^2}\left\{\ln\frac{4v^2}{3{,}17\omega_0^2 b^2(1-\beta^2)} - 1\right\}. \quad (21)$$

In deducing Eq. (21) we use the relation

$$\int x \operatorname{Im} \mu dx = \frac{\pi}{2}(\mu_0 - 1). \quad (22)$$

The logarithmic rise of the ionization losses follows from Eq. (21).

Taking into account the effect of polarization and combining the total ionization losses with the Cerenkov radiation losses we can neglect the damping in the final result. Then we have

$$W_b = \frac{e^2\omega_0^2}{2c^2}(\mu_0 - 1)\left\{\ln\frac{4v^2}{3{,}17\, b^2\omega_0^2} - \ln(1-\beta^2) - 1\right\}, \quad \beta < \frac{1}{\sqrt{\mu_0}}, \quad (23)$$

$$W_b = \frac{e^2\omega_0^2}{2c^2}(\mu_0 - 1)\left\{\ln\frac{4v^2}{3{,}17 b^2\omega_0^2} - \ln\beta^2(\mu_0 - 1) - \frac{1-\beta^2}{\beta^2(\mu_0-1)}\right\}, \beta > \frac{1}{\sqrt{\mu_0}}, \quad (24)$$

The total losses, as is easily seen from Eqs. (23) and (24), in contrast to Eq. (21), approach saturation at high velocities, while in the limiting case of ultrarelativistic motion the losses depend on the value of the coefficient of magnetic anisotropy. Actually, in the ultrarelativistic case Eq. (13) gives

$$W_b(v \sim c) = \frac{2\pi ne^4}{mv^2}\left(\frac{2k^0}{mc^2}\right)\ln\frac{mc^2}{3{,}17\pi b^2 e^2 n}\left(\frac{mc^2}{2k^0}\right). \quad (25)$$

Here n is the electron density of the medium, and $k^0 = k/n$ denotes a value which is the order of the energy of interaction of two elementary magnetic dipoles at a separation of the lattice constant, and in comparison with the rest energy of the electron is a negligibly small value $\sim 10^{-9}$.

The value of the spontaneous magnetization I_s of the ferromagnetic material does not occur in Eq. (14). Only the energy of magnetic anisotropy associated with the magnetic interaction of elementary magnetic dipoles plays an essential part for losses when $v/c \sim 1$ (see above).

Comparing Eq. (25) with the formula obtained by Fermi[4] for the energy losses in dielectrics,

$$W_b(v \sim c) = \frac{2\pi ne^4}{mv^2}\ln\frac{mc^2}{3{,}17\pi b^2 e^2 n}, \quad (26)$$

we see that it is possible to neglect the energy losses connected with the magnetic permeability in the ultrarelativistic case. This is particularly clear when, in the calculation of the losses according to Eq. (12), use is made of Eq. (11) together with the concrete dispersion formula

$$\varepsilon = 1 + \frac{4\pi ne^2}{m}\frac{1}{\omega_0^2 - \omega^2 - 2i\gamma'\omega}. \quad (27)$$

then

$$W_b(v \sim c) = \frac{4\pi ne^4}{mc^2}\left(1 + \frac{2k^0}{mc^2}\right) \ln\frac{mc^2}{3{,}17\pi b^2 e^2 n\left(1 + \frac{2k^0}{mc^2}\right)}. \quad (28)$$

Since a ferromagnetic material is usually a metal, it is expedient to consider the energy losses of a particle traversing a ferromagnetic material as losses in a conducting medium.

We note in conclusion that with the aid of Eq. (12) formulas may be obtained by an analogous method for the ionization and Cerenkov radiation losses both for a charged particle traversing a ferroelectric material and a magnetized particle traversing ferromagnetic material.

Translated by F. P. Dickey
50

[18] F. Bloch, Ann. Physik 16, 285 (1933)
[19] F. Bloch, Z. Phys. 81, 363 (1933)